S.A. ENGINEERING COLLEGE

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MATHEMATICS
PHYSICS
CHEMISTRY

FORMULAE

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MATHEMATICS

CHAPTER 1

APPLICATION OF MATRICES AND DETERMINANTS

THEOREM 1.1: For every square matrix A of order n, $A\,(\text{adj } A) = (\text{adj } A)\,A = |A|\,I_n$.

THEOREM 1.2: If a square matrix has an inverse, then it is unique.

THEOREM 1.3: Let A be square matrix of order n. Then A^{-1} exists if and only if A is non-singular.

THEOREM 1.4: If A is non-singular, then

(i) $|A^{-1}| = \dfrac{1}{|A|}$ (ii) $(A^T)^{-1} = (A^{-1})^T$

(iii) $(\lambda A)^{-1} = \dfrac{1}{\lambda} A^{-1}$, where λ is a non zero scalar.

THEOREM 1.5 (Left cancellation law):

Let A, B and C be square matrices of order n. If A is a non-singular and $AB = AC$, then $B = C$.

THEOREM 1.6 (Right cancellation law):

Let A, B and C be square matrices of order n. If A is non-singular and $BA = CA$, then $B = C$.

THEOREM 1.7 (Reversal law for inverse):

If A and B are non-singular matrices of the same order, then the product AB is also non-singular and $(AB)^{-1} = B^{-1} A^{-1}$.

THEOREM 1.8 (Law of double inverse):

If A is non-singular, then A^{-1} is also non-singular and $(A^{-1})^{-1} = A$.

THEOREM 1.9: If A is a non-singular square matrix of order n, then

(i) $(\text{adj } A)^{-1} = \text{adj } (A^{-1}) = \dfrac{1}{|A|} A$

(ii) $|\text{adj } A| = |A|^{n-1}$

(iii) $\text{adj } (\text{adj } A) = |A|^{n-2} A$

(iv) $(\text{adj } \lambda A) = \lambda^{n-1}$ adj (A) where λ is a non-zero scalar

(v) $|\text{adj } (\text{adj } A)| = |A|^{(n-1)^2}$

(vi) $(\text{adj } A)^T = \text{adj } (A^T)$

THEOREM 1.10: If A and B are any two non-singular square matrices of order n, then $\text{adj } (AB) = \text{adj } (B) \text{ adj } (A)$

Adjoint: $\text{adj } A = [A_{ij}]^T$

Inverse: $A^{-1} = \dfrac{1}{|A|}$ adj A; where $|A| \neq 0$

(i) $A^{-1} = \pm \dfrac{1}{\sqrt{\text{adj } A}} (\text{adj } A)$

(ii) $A^{-1} = \pm \dfrac{1}{\sqrt{\text{adj } A}} \text{ adj } (\text{adj } A)$

A matrix A is orthogonal if $AA^T = A^T A = I$

A matrix A is a orthogonal if and only if A is a non-singular and $A^{-1} = A^T$

Methods to solve the system of linear equations $AX = B$

(i) Matrix inversion method: $X = A^{-1} B$, $|A| \neq 0$

(ii) By Cramer's rule $x = \dfrac{\Delta_1}{\Delta}$, $y = \dfrac{\Delta_2}{\Delta}$, $z = \dfrac{\Delta_3}{\Delta}$, $\Delta \neq 0$

Gaussian elimination method

Rank method

$\rho(A) = \rho([A\|B]) = 3$	Consistent	One solution
$\rho(A) = \rho([A\|B]) < 3$	Consistent	Many solution
$\rho(A) \neq \rho([A\|B])$	Inconsistent	No solution

The homogenous system of linear equation A X = 0

Has a trivial solution $|A| \neq 0$

Has a non-trivial solution, $|A| = 0$

CHAPTER 2

COMPLEX NUMBERS

Definition:

A complex number is of the form $x + iy$ (or $x + yi$) where x and y are real numbers. x is called the real part and y is called the imaginary part of the complex number.

Properties:

Let $z = a + ib$ then $\bar{z} = a - ib$

✶ $z\bar{z} = (a + ib)(a - ib) = a^2 + b^2$

✶ conjugate of $\bar{z}$ is z *i.e*, $\bar{\bar{z}} = z$

✶ $a = \text{Re}(z) = \dfrac{z + \bar{z}}{2}$

$b = \text{Im}(z) = \dfrac{z - \bar{z}}{2i}$

✶ The modulus or absolute value of z

$|z| = \sqrt{a^2 + b^2}$

$z\bar{z} = |z|^2$ $\text{Re}(z) \leq |z|$ $\text{Im}(z) \leq |z|$

Notes:

- Triangle inequality
    - $|z_1 + z_2| \le |z_1| + |z_2|$
    - $|z_1 - z_2| \le |z_1| + |z_2|$
    - $|z_1 - z_2| \ge |z_1| - |z_2|$
- The argument of z is $\theta = \tan^{-1}\dfrac{y}{x}$
- $|z_1 z_2| = |z_1| \cdot |z_2|$

  $\arg(z_1 \cdot z_2) = \arg z_1 + \arg z_2$
- $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z2|}, \qquad (z_2 \ne 0)$

  $\arg\left(\dfrac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

Euler's formula:

The symbol $e^{i\theta}$ or $\exp(i\theta)$

$e^{i\theta} = \cos\theta + i\sin\theta$

De Moivre's theorem:

- $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
- $(\cos\theta + i\sin\theta)^{\frac{p}{q}} = \cos\dfrac{p}{q}\theta + i\sin\dfrac{p}{q}\theta$
- $(\cos\theta + i\sin\theta)^{-1} = \cos n\theta - i\sin n\theta$

Results:

- $w^n = 1, \qquad 1 + w + w^2 = 0$ and $w^3 = 1$
- sum of the roots is 0
- The roots are in G.P. with common ratio w.
- The arguments are in A.P. with common difference $\dfrac{2\pi}{n}$
- Product of the roots = $(-1)^{n+1}$
- Working rule to find the nth roots of a complex number:

  Step 1: Write the given number in polar form

  Step 2: Add $2k\pi$ to the argument

  Step 3: apply De Moivre's theorem (bring the power to inside)

  Step 4: put $k = 0, 1...$ upto $n-1$

cube roots of unity : $(1)^{\frac{1}{3}} = \left(\cos\dfrac{2k\pi}{3} + i\sin\dfrac{2k\pi}{3}\right)$

where $k = 0, 1, 2$

Powers of imaginary unit i

$i^0 = 1;\ i^1 = i$	$i^2 = -1$
$(i)^{-1} = \dfrac{1}{i} = \dfrac{i}{(i)^2} = -i$	$(i)^{-2} = \dfrac{1}{i^2} = -1$

$i^3 = i^2\, i = -i$	$i^4 = i^2\, i^2 = 1$
$(i)^{-3} = i$	$(i)^{-4} = 1 = i^4$

$i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

PROPERTY 1 (The commutative property under addition):

For all complex numbers z_1 and z_2, prove that $z_1 + z_2 = z_2 + z_1$

PROPERTY 2 (Inverse property under multiplication):

The multiplicative inverse of a nonzero complex number $z = x + iy$, is $\dfrac{x}{x^2 + y^2} - i\dfrac{y}{x^2 + y^2}$

PROPERTY 3: For all two complex numbers z_1 and z_2, prove that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

PROPERTY 4: $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$ where x_1, x_2, y_1 and $y_2 \in \mathbb{R}$

PROPERTY 5: z is purely imaginary if and only if $z = -\overline{z}$

PROPERTY 6 (Triangle inequality):

For any two complex number z_1 and z_2, prove that $|z_1 + z_2| \le |z_1| + |z_2|$

PROPERTY 7:

For any complex number z_1 and z_2, prove that $|z_1 z_2| = |z_1|\,|z_2|$

PROPERTY 8:

If $z = r(\cos\theta + i\sin\theta)$, then $z^{-1} = \dfrac{1}{r}(\cos\theta - i\sin\theta)$

PROPERTY 9:

If $z_1 = r_1(\cos\theta + i\sin\theta)$ and $z_2 = r_2(\cos\theta + i\sin\theta)$ then $z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

PROPERTY 10:

If $z_1 = r_1(\cos\theta + i\sin\theta)$ and $z_2 = r_2(\cos\theta + i\sin\theta)$ then

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$$

Notes:

PROPERTIES OF COMPLEX CONJUGATES:

If $z = x + iy$ then $\bar{z} = x - iy$

(1) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

(2) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

(3) $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$

(4) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}; \overline{z_2} \neq 0$

(5) $\operatorname{Re}(z) = \frac{z + \bar{z}}{2}$

(6) $\operatorname{Im}(z) = \frac{z - \bar{z}}{2i}$

(7) $\overline{(z^n)} = (\bar{z})^n$, where n is an integer

(8) If z is real if an only if $z = \bar{z}$

(9) If z is purely imaginary if an only if $z = -\bar{z}$

(10) $\bar{\bar{z}} = z$

PROPERTIES OF MODULUS OF A COMPLEX NUMBER

(1) $|z| = |\bar{z}|$

(2) $|z_1 + z_2| \le |z_1| + |z_2|$ (Triangle inequality)

(3) $|z_1 z_2| = |z_1|\,|z_2|$

(4) $|z_1 - z_2| \ge |z_1| - |z_2|$

(5) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

(6) $|z^n| = |z|^n$, where n is an integer

(7) $\operatorname{Re}(z) \le |z|$

(8) $\operatorname{Im}(z) \le |z|$

SQUARE ROOT:

$$\sqrt{a+ib} = \pm\left(\sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right)$$

POLAR FORM: $z = r(\cos\theta + i\sin\theta)$

GENERAL RULE FOR DETERMINING ARGUMENT:

SECOND QUADRANT		FIRST QUADRANT	
sin θ +ive	$\theta = \pi - a$	$\theta = a$	sin θ +ive
cos θ –ive			cos θ +ive
sin θ –ive	$\theta = -\pi + a$	$\theta = -a$	sin θ –ive
cos θ –ive			cos θ +ive
THIRD QUADRANT		**FOURTH QUADRANT**	

nth roots of complex numbers:

$$z^{1/n} = r^{1/n}\left(\cos\left(\frac{\theta + 2k\pi}{n}\right) + i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right)$$

CHAPTER 3

THEORY OF EQUATIONS

Quadratic equations:

For the quadratic equation $ax^2 + bx + c = 0$

$b^2 - 4ac$ is called the discriminal. It denotes by Δ

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We also learnt that

$\Delta = 0$ if and onlyif the roots are equal

$\Delta > 0$ if and onlyif the roots are distinct

$\Delta < 0$ if and onlyif the quadratic equation has no real roots.

Vieta's formula for Quadratic equations:

Let α and β be the roots of the quadratic equation. $ax^2 + bx + c = 0$

Sum of roots $= \alpha + \beta = -\frac{b}{a}$

Product of roots $= \alpha / \beta = \frac{c}{a}$

Vieta's formula for polynomial equations of degree 2

$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$ that is a quadratic equation with given root is x^2 – (sum of the roots) x + (products of the roots) = 0

Vieta's formula for polynomial equations of degree 3

$\Rightarrow x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0.$

Vieta's formula for polynomial equations of degree $n > 3$

$\Sigma\alpha = \alpha + \beta + \gamma + \delta$

$\Sigma\alpha\beta = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

$\Sigma\alpha\beta\gamma = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

$\Sigma\alpha\beta\gamma\delta = \alpha\beta\gamma\delta$

Notes:

THE FUNDAMENTAL THEOREM OF ALGEBRA:

A polynomial of degree $n \geq 1$ has at least one root in C.

COMPLEX CONJUCATE ROOT THEOREM:

Imaginary (non-real complex) roots occur as conjugate pairs, if the coefficients of the polynomial are real

RATIONAL ROOT THEOREM:

- Let $a_n x^n + \ldots + a_1 x + a_0 = 0$ with $a_n \neq 0$ and $a_0 \neq 0$ be a polynomial with integer coefficients.
- If p/q, with $(p, q) = 1$, is a root of the polynomial, then p is a factor of a_0 and q is a factor of a_n.
- Methods to solve some special types of polynomial equations like polynomials having only even powers , partly factored polynomials , polynomials with sum of the coefficients is zero, reciprocal equations.

DESCARTES RULE: $s - p$

If p is the number of positive roots of a polynomial $p(x)$ and s is the number of sign changes in coefficients of $p(x)$, then $s - p$ is a nonnegative even integer.

CHAPTER 4

INVERSE TRIGONOMETRIC FUNCTIONS

PROPERTY 1:

(i) $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos\theta) = \theta$, if $\theta \in [0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta$, if $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$, if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$

(v) $\sec^{-1}(\sec\theta) = \theta$, if $\theta \in [0, \pi] \setminus \left\{\frac{\pi}{2}\right\}$

(vi) $\cot^{-1}(\cot\theta) = \theta$, if θ if $0 \in (0, \pi)$

PROPERTY 2:

(i) $\sin(\sin^{-1} x) = x$, if $x \in [-1, 1]$

(ii) $\cos(\cos^{-1} x) = x$, if $x \in [-1, 1]$

(iii) $\tan(\tan^{-1} x) = x$, if $x \in \mathbb{R}$

(iv) $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x$, if $x \in \mathbb{R} \setminus (-1, 1)$

(v) $\sec(\sec^{-1} x) = x$, if $x \in \mathbb{R} \setminus (-1, 1)$

(vi) $\cot(\cot^{-1} x) = x$, if $x \in \mathbb{R}$

PROPERTY 3 (RECIPROCAL INVERSE IDENTITIES):

(i) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec} x$, if $x \in \mathbb{R} \setminus (-1, 1)$

(ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec x$, if $x \in \mathbb{R} \setminus (-1, 1)$

(iii) $$\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x & , \text{if } x > 0 \\ -\pi + \cot^{-1} x & , \text{if } x < 0 \end{cases}$$

PROPERTY 4 (REFLECTION IDENTITIES):

(i) $\sin^{-1}(-x) = -\sin^{-1} x$, if $x \in [-1, 1]$

(ii) $\tan^{-1}(-x) = -\tan^{-1} x$, if $x \in \mathbb{R}$

(iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$, if $|x| \geq 1$ or $x \in \mathbb{R} \setminus (-1, 1)$

(iv) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, if $x \in [-1, 1]$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1} x$, if $|x| \geq 1$ or $x \in \mathbb{R} \setminus (-1, 1)$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x$, if $x \in \mathbb{R}$

PROPERTY 5 (CO FUNCTION INVERSE IDENTITIES:

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, x \in [-1, 1]$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, if $x \in \mathbb{R}$

(iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$, if $|x| \geq 1$ or $x \in \mathbb{R} \setminus (-1, 1)$

PROPERTY 6:

(i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$,

where either $x^2 + y^2 \leq 1$ or $xy < 0$

(ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$,

where either $x^2 + y^2 \leq 1$ or $xy > 0$

(iii) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$,

if $x + y \geq 0$

(iv) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$, if $x \leq y$

(v) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$

(vi) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $xy > -1$

Notes:

PROPERTY 7:

(i) $2\tan^{-1}x = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), |x|<1$

(ii) $2\tan^{-1}x = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right), x\ge 0$

(iii) $2\tan^{-1}x = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right), |x|\le 1$

PROPERTY 8:

(i) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x,$

$\quad$ if $|x| \le \dfrac{1}{\sqrt{2}}$ or $-\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}}$

(ii) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x,$ if $\dfrac{1}{\sqrt{2}} \le x \le 1$

PROPERTY 9:

(i) $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$, if $0 \le x \le 1$

(ii) $\sin^{-1}x = -\cos^{-1}\sqrt{1-x^2}$, if $-1 \le x < 0$

(iii) $\sin^{-1}x = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$, if $-1 < x < 1$

(iv) $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$, if $0 \le x \le 1$

(v) $\cos^{-1}x = \pi - \sin^{-1}\sqrt{1-x^2}$, if $-1 \le x < 0$

(vi) $\tan^{-1}x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$, if $x > 0$

PROPERTY 10:

(i) $3\sin^{-1}x = \sin^{-1}(3x-4x^3), x \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

(ii) $3\cos^{-1}x = \cos^{-1}(4x^3-3x), x \in \left[\dfrac{1}{2}, 1\right]$

CHAPTER 5

TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

THEOREM 1:

The circle passing through the points of intersection of the line $lx + my + n = 0$ and the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ the circle of the form $x^2 + y^2 + 2gx + 2fy + c + \lambda\,(lx + my + n) = 0, \lambda \in \mathbb{R}^1$

THEOREM 2:

The equation of a circle with (x_1, y_1) and (x_2, y_2) as extremities of one of the diameters of the circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

THEOREM 3:

The position of a point $P(x_1, y_1)$ with respect to a given circle $x^2 + y^2 + 2gx + 2fy + c = 0$ in the plane containing the circle is outside or on or inside the circle according as

$$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c \text{ is } \begin{cases} > & 0, \text{ or} \\ = & 0, \text{ or} \\ < & 0, \end{cases}$$

THEOREM 4: From any point outside the circle $x^2 + y^2 = a^2$ two tangent can be drawn.

THEOREM 5: The sum of the focal distances of any point on the ellipse is equal to length of the major axis.

THEOREM 6: Three normal can be drawn to a parabola $y^2 = 4ax$ from a given point, one of which is always real.

TANGENT AND NORMAL:

CURVE: CIRCLE

EQUATION: $x^2 + y^2 = a^2$

EQUATION OF TANGENT:

(i) Cartesian form: $xx_1 + yy_1 = a^2$

(ii) Parametric form: $x\cos\theta + y\sin\theta = a$

EQUATION OF NORMAL:

(i) Cartesian form: $xy_1 - yx_1 = 0$

(ii) Parametric form: $x\sin\theta - y\cos\theta = 0$

CURVE: PARABOLA

EQUATION: $y^2 = 4ax$

EQUATION OF TANGENT:

(i) $yy_1 = 2a\,(x + x_1)$

(ii) $yt = x + at^2$

EQUATION OF NORMAL:

(i) $xy_1 + 2y = 2ay_1 + x_1 y_1$

(ii) $y + xt = at^3 + 2at$

CURVE: ELLIPSE

EQUATION: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

EQUATION OF TANGENT:

(i) $\dfrac{x\,x_1}{a^2} + \dfrac{y\,y_1}{b^2} = 1$

Notes:

(ii) $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$

EQUATION OF NORMAL:

(i) $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2-b^2$

(ii) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

CURVE: HYPERBOLA

EQUATION: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

EQUATION OF TANGENT:

(i) $\frac{x\,x_1}{a^2}-\frac{y\,y_1}{b^2}=1$

(ii) $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$

EQUATION OF NORMAL:

(i) $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

(ii) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

CONDITION FOR THE SINE $y = mx + c$ **TO BE A TANGENT TO THE CONICS**

CURVE: CIRCLE

EQUATION: $x^2 + y^2 = a^2$

CONDITION TO BE TANGENT: $c^2 = a^2\,(l + m^2)$

POINT OF CONTACT: $\left(\frac{\mp am}{\sqrt{1+m^2}},\frac{\pm a}{\sqrt{1+m^2}}\right)$

EQUATION OF TANGENT: $y = mx \pm \sqrt{1+m^2}$

CURVE: PARABOLA

EQUATION: $y^2 = 4ax$

CONDITION TO BE TANGENT: $c=\frac{a}{m}$

POINT OF CONTACT: $\left(\frac{a}{m^2},\frac{2a}{m}\right)$

EQUATION OF TANGENT: $y = mx+\frac{a}{m}$

CURVE: ELLIPSE

EQUATION: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

CONDITION TO BE TANGENT: $c^2 = a^2m^2+b^2$

POINT OF CONTACT: $\left(\frac{-a^2m}{c},\frac{b^2}{c}\right)$

EQUATION OF TANGENT: $y = mx \pm \sqrt{a^2m^2+b^2}$

CURVE: HYPERBOLA

EQUATION: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

CONDITION TO BE TANGENT: $c^2 = a^2m^2-b^2$

POINT OF CONTACT: $\left(\frac{-a^2m}{c},\frac{-b^2}{c}\right)$

EQUATION OF TANGENT: $y = mx \pm \sqrt{a^2m^2-b^2}$

PARAMETRIC FORMS

CONIC: CIRCLE

PARAMETRIC EQUATIONS: $x = a\cos\theta;\ y = a\sin\theta$

PARAMETER: θ

RANGE OF PARAMETER: $0 \le \theta < 2\pi$

ANY POINT ON THE CONIC: ‘θ’ or $(a\cos\theta, b\sin\theta)$

CONIC: PARABOLA

PARAMETRIC EQUATIONS: $x = at^2;\ y = 2at$

PARAMETER: t

RANGE OF PARAMETER: $-\infty < t < \infty$

ANY POINT ON THE CONIC: ‘t’ or $(at^2, 2at)$

CONIC: ELLIPSE

PARAMETRIC EQUATIONS: $x = a\cos\theta;\ y = b\sin\theta$

PARAMETER: θ

RANGE OF PARAMETER: $0 \le \theta < 2\pi$

ANY POINT ON THE CONIC: ‘θ’ or $(a\cos\theta, b\sin\theta)$

CONIC: HYPERBOLA

PARAMETRIC EQUATIONS: $x = a\sec\theta;\ y = b\tan\theta$

PARAMETER: θ

RANGE OF PARAMETER: $-\pi \le \theta \le \pi$ Except $\theta = \pm\frac{\pi}{2}$

Notes:

ANY POINT ON THE CONIC: 'θ' or $(a \sec\theta, b \tan\theta)$

PARABOLA

EQUATION: $(y - k)^2 = 4a(x - h)$

VERTICES: (h, k)

FOCUS: $(h + a, 0 + k)$

AXIS OF SYMMETRY: $y = k$

EQUATION OF DIRECTRIX: $x = h - a$

LENGTH OF LATUS RECTUM: $4a$

EQUATION: $(y - k)^2 = -4a(x - h)$

VERTICES: (h, k)

FOCUS: $(h - a, 0 + k)$

AXIS OF SYMMETRY: $y = k$

EQUATION OF DIRECTRIX: $x = h + a$

LENGTH OF LATUS RECTUM: $4a$

EQUATION: $(x - h)^2 = 4a(y - k)$

VERTICES: (h, k)

FOCUS: $(0 + h, a + k)$

AXIS OF SYMMETRY: $x = h$

EQUATION OF DIRECTRIX: $y = k - a$

LENGTH OF LATUS RECTUM: $4a$

EQUATION: $(x - h)^2 = -4a(y - k)$

VERTICES: (h, k)

FOCUS: $(0 + h, -a + k)$

AXIS OF SYMMETRY: $x = h$

EQUATION OF DIRECTRIX: $y = k + a$

LENGTH OF LATUS RECTUM: $4a$

PARAMETRIC FORMS

Identifying the conic from the general equation of conic

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

The graph of the second degree equation is one of a circle, parabola, an ellipse, a hyperbola, a point, an empty set, a single line or a pair of lines. When,

(1) $A = C = 1$, $B = 0$, $D = -2h$, $E = -2k$, $F = h^2 + k^2 - r^2$ the general equation reduces to $(x - h)^2 + (y - h)^2 = r^2$, which is a circle .

(2) $B = 0$ and either A or $C = 0$, the general equation yields a parabola under study, at this level

(3) $A \neq C$ and A and C are of the same sign the general equation yields an ellipse.

(4) $A \neq C$ and A and C are of the opposite signs the general equation yields a hyperbola

Types of ellipses with centre at (*h*, *k*)

EQUATION	CENTRE	MAJOR AXIS	VERTICES	FOCI
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $a^2 > b^2$ a) Major axis parallel to the x-axis Foci are c units right and c units left of centre, where $c^2 = a^2 - b^2$	(h, k)	Parallel to the x-axis	$(h - a, k)$ $(h + a, k)$	$(h - c, k)$ $(h + c, k)$
$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ $a^2 > b^2$ b) Major axis parallel to the y-axis Foci are c units right and c units left of centre , where $c^2 = a^2 - b^2$	(h, k)	Parallel to the y-axis	$(h, k - a)$ $(h, k + a)$	$(h, k - c)$ $(h, k + c)$

Types of Hyperbola with centre at (*h*, *k*)

(a) Transverse axis parallel to the x-axis	(a) Transverse axis parallel to the x-axis The equation of a hyperbola with centre $C(h, k)$ and transverse axis parallel to the x-axis is given by $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2}$ The coordinates of the vertices are $A(h + a, k)$ and $A'(h - a, k)$. The coordinates of the foci are $S(h + c, k)$ and $S'(h - c, k)$ where $c^2 = a^2 + b^2$ The equations of directrices are $x = \pm\frac{a}{e}$

Notes:

(b) Transverse axis parallel to the *y*-axis	(b) Transverse axis parallel to the *y*- axis. The equation of a hyperbola with centre $C(h, k)$ and transverse axis parallel to the *y*-axis is given by $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1.$ The coordinates of the vertices are $A(h, k + a)$ and $A'(h, k - a)$, the coordinates of the foci are $S(h, k+c)$ and $S'(h, k - c)$ where $c^2 = a^2 + b^2$ The equations of directrices are $y = \pm\frac{a}{e}$

CHAPTER 6

APPLICATIONS OF VECTOR ALGEBRA

THEOREM 1:

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}, \text{ then } (\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

THEOREM 2:

For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $(\vec{a}\times\vec{b})\cdot\vec{c} = \vec{a}\cdot(\vec{b}\times\vec{c})$

THEOREM 3:

The scalar triple product preserves addition and scalar multiplication. That is,

$$[(\vec{a}+\vec{b}), \vec{c}, \vec{d}] = [\vec{a}, \vec{c}, \vec{d}] + [\vec{b}, \vec{c}, \vec{d}];$$

$$[\lambda\,\vec{a}, \vec{b}, \vec{c}] = \lambda[\vec{a}, \vec{b}, \vec{c}], \forall\lambda\in\mathbb{R}$$

$$[\vec{a}, (\vec{b}+\vec{c}), \vec{d}] = [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}];$$

$$[\vec{a}, \lambda\vec{b}, \vec{c}] = \lambda[\vec{a}, \vec{b}, \vec{c}], \forall\lambda\in\mathbb{R}$$

$$[\vec{a}, \vec{b}, (\vec{c}+\vec{d})] = [\vec{a}, \vec{b}, \vec{c}] + [\vec{a}, \vec{b}, \vec{d}];$$

$$[\vec{a}, \vec{b}, \lambda\vec{c}] = \lambda[\vec{a}, \vec{b}, \vec{c}], \forall\lambda\in\mathbb{R}$$

THEOREM 4:

The scalar triple product of three non-zero vectors is zero, if and only if the three vector are coplanar

THEOREM 5:

Three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if, there exist scalars $r, s, t \in \mathbb{R}$ such that at least one of them is non-zero and $r\vec{a} + s\vec{b} + t\vec{c} = 0$

THEOREM 6:

If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{p}, \vec{q}, \vec{r}$ are two system of three vectors, and if

$$\vec{p} = x_1\vec{a} + y_1\vec{b} + z_1\vec{c}, \text{ and}$$

$$\vec{q} = x_2\vec{a} + y_2\vec{b} + z_2\vec{c}$$

$$\vec{r} = x_3\vec{a} + y_3\vec{b} + z_3\vec{c}, \text{ then}$$

$$[\vec{p}, \vec{q}, \vec{r}] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}]$$

THEOREM 7:

The vector triple product satisfies the following properties:

$$(\overrightarrow{a_1}+\overrightarrow{a_2})\times(\overrightarrow{b_1}\times\overrightarrow{c_1}) = \overrightarrow{a_1}\times(\vec{b}\times\vec{c}) + \overrightarrow{a_2}\times(\vec{b}\times\vec{c}),$$

$$(\lambda\vec{a})\times(\vec{b}\times\vec{c}) = \lambda\,(\vec{a}\times(\vec{b}\times\vec{c})), \lambda\in\mathbb{R}$$

$$\vec{a}\times((\overrightarrow{b_1}+\overrightarrow{b_2})\times\vec{c}) = \vec{a}\times(\overrightarrow{b_1}\times\vec{c}) + \vec{a}\times(\overrightarrow{b_2}\times\vec{c}),$$

$$\vec{a}\times((\lambda\vec{b})\times\vec{c}) = \lambda\,(\vec{a}\times(\vec{b}\times\vec{c})), \lambda\in\mathbb{R}$$

$$\vec{a}\times(\overrightarrow{b_1}\times(\overrightarrow{c_1}+\overrightarrow{c_2})) = \vec{a}\times(\vec{b}\times\overrightarrow{c_1}) + \vec{a}\times(\vec{b}\times\overrightarrow{c_2}),$$

$$\vec{a}\times(\vec{b}\times(\lambda\,\vec{c})) = \lambda\,(\vec{a}\times(\vec{b}\times\vec{c})), \lambda\in\mathbb{R}$$

THEOREM 8:

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ we have

$$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c}$$

THEOREM 9 (JACOBI'S IDENTITY):

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ we have

$$\vec{a}\times(\vec{b}\times\vec{c}) + \vec{b}\times(\vec{c}\times\vec{a}) + \vec{c}\times(\vec{a}\times\vec{b}) = \vec{0}$$

THEOREM 10 (LAGRANGE'S IDENTITY):

For any four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ we have

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d} \\ \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{d} \end{vmatrix}$$

THEOREM 11:

The vector equation of a straight line passing through a fixed point with position vector $\vec{a}$ and parallel to a given

Notes:

vector $\vec{b}$ is $\vec{r} = \vec{a} + t\vec{b}$ where $t \in \mathbb{R}$

THEOREM 12:

The parametric form of of vector equation of a line passing through two given points vector are $\vec{a}$ and $\vec{b}$ respectively is

$\vec{r} = \vec{a} + \vec{t}\,(\vec{b} - \vec{a})$ where $t \in \mathbb{R}$

Two lines are said to be **a coplanar** if their lie in the same plane.

Two lines in space are called **skew lines** if they are not parallel and do not intersect

THEOREM 13:

The shortest distance between the two parallel lines

$\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{b}$ is given by

$d = \dfrac{|(\vec{c} - \vec{a}) \times \vec{b}|}{|\vec{b}|}$ where $|\vec{b}| \neq 0$

THEOREM 14:

The shortest distance between the two skew lines

$\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$ is given by

$\delta = \dfrac{|(\vec{c} - \vec{a}) \times (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|}$ where $|\vec{b} \times \vec{d}| \neq 0$

THEOREM 15:

The equation of the plane at a distance ρ from the origin and perpendicular to the unit normal vector $\hat{d}$ is $\vec{r} \cdot \hat{d} = p$

THEOREM 16:

The general equation $ax + by + cz + d = 0$ of first degree in x, y, z represents a plane.

THEOREM 17:

If three non-collinear points with position vectors $\vec{a}, \vec{b}, \vec{c}$ are given, then the vector equation of the plane passing through the given points in parametric form is $\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t(\vec{c} - \vec{a})$ where $\vec{b} \neq \vec{0}, \vec{c} \neq \vec{0}$ and $s, t \in \mathbb{R}$

THEOREM 18:

The acute angle θ between the two plans

$\vec{r} \cdot \overrightarrow{n_1} = p_1$ and $\vec{r} \cdot \overrightarrow{n_2} = p_2$

is given by $\theta = \cos^{-1}\left(\dfrac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}\right)$

THEOREM 19:

The acute angle θ between the two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\theta = \cos^{-1}\left(\frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$$

THEOREM 20:

The perpendicular distance from a point with position vector $\vec{u}$ to the plane $\vec{r}\,.\,\vec{n} = p$ is given by $\delta = \dfrac{|\vec{u}\,.\,\vec{n} - p|}{|\vec{n}|}$

THEOREM 21:

The distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by $\dfrac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

THEOREM 22:

The vector equation of a plane which passes through the line of intersection of the planes

$\vec{r}\,.\,\overrightarrow{n_1} = d_1$ and $\vec{r}\,.\,\overrightarrow{n_2} = d_2$ is given by

$(\vec{r}\,.\,\overrightarrow{n_1} - d_1) + \lambda(\vec{r}\,.\,\overrightarrow{n_2} - d_2) = 0$ where $\lambda \in \mathbb{R}$

THEOREM 23:

The position vector of the point of intersection of the straight line $\vec{r} = \vec{a} + t\vec{b}$ and the plane $\vec{r}\,.\,\overrightarrow{n_1} = p$ is $\vec{a} + \left(\dfrac{p - (\vec{a} \cdot \vec{n})}{\vec{b} \cdot \vec{n}}\right)$

provided $\vec{b} \cdot \vec{n} \neq 0$

The shortest distance between the two skew lines is the length of the line segment perpendicular to both the skew lines.

A straight line which is perpendicular to a plane is called a normal to the plane

Notes:

PROPERTIES OF SCALAR AND VECTOR PRODUCTS

SCALAR PRODUCT	VECTOR PRODUCT
$\vec a\cdot\vec b=\lvert\vec a\rvert\lvert\vec b\rvert\cos\theta$	$\vec a\times\vec b=\lvert\vec a\rvert\lvert\vec b\rvert\sin\theta\,\hat n$
$\vec a\cdot\vec b=0$ (i) $\vec a$ is zero vector $\vec b$ any other vector (ii) $\vec b$ is zero vector $\vec a$ any other vector $\vec a$ and $\vec b$ are perpendicular	$\vec a\times\vec b=0$ (i) $\vec a$ is zero vector $\vec b$ any other vector (ii) $\vec b$ is zero vector $\vec a$ any other vector $\vec a$ and $\vec b$ are parallel
$\vec a\cdot\ \vec a=a^2$	$\vec a\times\vec a=\vec 0$
$\vec i\cdot\vec i=\vec j\cdot\vec j=\vec k\cdot\vec k=1$ $\vec i\cdot\vec j=\vec j\cdot\vec k=\vec k\cdot\vec i=0$	$\vec i\times\vec i=\vec j\times\vec j=\vec k\times\vec k=\vec 0$ $\vec i\times\vec j=\vec k;\ \vec j\times\vec k=\vec i;\ \vec k\times\vec i=\vec j$ $\vec i\times\vec k=-\vec j;\ \vec j\times\vec i=-\vec k;\ \vec k\times\vec j=-\vec i$
$\vec a=a_1\vec i+a_2\vec j+a_3\vec k$ $\vec b=b_1\vec i+b_2\vec j+b_3\vec k$ $\vec a\cdot\vec b=(a_1b_1+a_2b_2+a_3b_3)$	$\vec a=a_1\vec i+a_2\vec j+a_3\vec k$ $\vec b=b_1\vec i+b_2\vec j+b_3\vec k$ $\vec a\times\vec b=\begin{vmatrix}\vec i & \vec j & \vec k\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$
SCALAR PRODUCT	**VECTOR PRODUCT**
$\theta=\cos^{-1}\left(\dfrac{\vec a\cdot\vec b}{\lvert\vec a\rvert\lvert\vec b\rvert}\right)$	$\theta=\sin^{-1}\left(\dfrac{\lvert\vec a\times\vec b\rvert}{\lvert\vec a\rvert\lvert\vec b\rvert}\right)$

A straight will lie on a plane of every point on the line , lie in the plane and the normal to the plane is perpendicular to the line .

CHAPTER 7

APPLICATION OF DIFFERENTIAL CALCULUS

DIFFERENTIAL CALCULUS BASIC FORMULAS

1	$\dfrac{d}{dx}[x^n]=nx^{n-1}$	2	$\dfrac{d}{dx}(x)=x$
3	$\dfrac{d}{dx}[\log x]=\dfrac{1}{x}$	4	$\dfrac{d}{dx}[c]=0$
5	$\dfrac{d}{dx}[e^x]=e^x$	6	$\dfrac{d}{dx}[\sin x]=\cos x$
7	$\dfrac{d}{dx}[\cos x]=-\sin x$	8	$\dfrac{d}{dx}[\tan x]=\sec^2 x$
9	$\dfrac{d}{dx}[\sec x]=\sec x\tan x$	10	$\dfrac{d}{dx}[\cos ec\,x]=-\cos ec\,x\cot x$
11	$\dfrac{d}{dx}[\cot x]=-\cos ec^2 x$	12	$\dfrac{d}{dx}[\sin^{-1}x]=\dfrac{1}{\sqrt{1-x^2}}$
13	$\dfrac{d}{dx}[\cos^{-1}x]=\dfrac{-1}{\sqrt{1-x^2}}$	14	$\dfrac{d}{dx}[\tan^{-1}x]=\dfrac{1}{1+x^2}$
15	$\dfrac{d}{dx}[\cot^{-1}]=\dfrac{-1}{1+x^2}$	16	$\dfrac{d}{dx}[\sec^{-1}x]=\dfrac{1}{x\sqrt{x^2-1}}$
17	$\dfrac{d}{dx}[\cos ec^{-1}\,x]=\dfrac{-1}{x\sqrt{x^2-1}}$	18	$\dfrac{d}{dx}(u+v)=\dfrac{d}{dx}(u)+\dfrac{d}{dx}(v)$
19	$\dfrac{d}{dx}(uv)=u\dfrac{dv}{dx}+\dfrac{du}{dx}v$	20	Quotient Rule $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$
21	Chain Rule $\dfrac{d}{dx}(f(g(x))=f'(g(x))g'(x)$	22	$\dfrac{d}{dx}(kx)=k$
23	$\dfrac{d}{dx}(\sqrt{x})=\dfrac{1}{2\sqrt{x}}$		

VELOCITY & ACCELERATION

If distance $x=f(t)$

Velocity $v=f'(t)$ or $\dfrac{dx}{dt}$,

acceleration $a=\dfrac{dv}{dt}=f''(t)$ or $\dfrac{d^2x}{dt^2}$

(i) Initial velocity means velocity at $t=0$

(ii) Initial acceleration means acceleration at $t=0$

(iii) If the motion is upward, at the maximum height the velocity is zero.

Notes:

(iv) If the motion is horizontal $v = 0$ when the particle cones to rest.

TANGENT $y - y_1 = m(x - x_1)$

NORMAL $y - y_1 = \frac{-1}{m}(x - x_1)$

If the two curves are parallel at (x_1, y_1) then $m_1 = m_2$

If the two curves are perpendicular at (x_1, y_1), then $m_1 m_2 = -1$

ANGLE BETWEEN TWO CURVES

$$\tan \varphi = \left|\frac{m_1 - m_2}{1 + m_1\, m_2}\right|$$

ROLLE'S THEOREM

Let $f(x)$ be continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b) let $f(a) = f(b)$, then there is at least one point $c \in (a, b)$ where $f'(c) = 0$

LAGRANGE'S MEAN VALUE THEOREM

Let $f(x)$ be continuous in a closed interval $[a, b]$ and differentiable on the open interval (a, b) let $f(a) = f(b)$, then there exist at least one point $c \in (a, b)$ where

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

TAYLOR THEOREM

$$f(x) = f(a) + \frac{f'(a)}{1!}(x - a) + \cdots + \frac{f^n(a)}{n!}(x - a)^n + \cdots.$$

MACLAURIN'S THEOREM

$$f(x) = f(0) + \frac{f'(0)}{1!}(x) + \cdots + \frac{f^n(0)}{n!}(x)^n + \cdots.$$

WORKING RULES

CRITICAL NUMBERS	STATIONARY POINT	INCREASING & DECREASING
If $f(x)$ given (i) Find $f'(x)$ (ii) Solve $f'(x) = 0$ and get critical number (CN) (If degree of $f(x)$ is n. there are $n - 1$ critical numbers are possible)	If $f(x)$ given (i) Find $f'(x)$ (ii) Solve $f'(x) = 0$ and get C.N (iii) Put C.N in $f(x)$ and get stationary points (S.P). (if degree of $f(x)$ is n. there are $n - 1$ stationary points are possible)	If $f(x)$ given (i) Find $f'(x)$ (ii) Solve $f'(x) = 0$ and get C.N (iii) Fix the limits on both sides of C.N (iv) Check the sign of $f'(x)$ in above limits If $f'(x) > 0$ increasing If $f'(x) < 0$ decreasing

WORKING RULES

MONOTONICITY	ABSOLUTE MAXIMUM/ MINIMUM IN [A , B]	CONCAVITY / CONVEXITY
If $f(x)$ given (i) Find $f'(x)$ (ii) Solve $f'(x) = 0$ and get C.N (iii) Fix the limits on both sides of C.N (iv) Check the sign of $f'(x)$ in above limits If $f'(x) > 0$ Increasing If $f'(x) < 0$ Decreasing both sides of C.N If $f'(x)$ has same sign then it is Monotonicity	If $f(x)$ given (i) Find $f'(x)$ (ii) Solve $f'(x) = 0$ and get C.N (iii) Put the values of end points $[a, b]$ in $f(x)$ i.e., find $f(a)$ & $f(b)$ and also find value of $f(x)$ in C.N Compare all Maximum one is absolute maximum Minimum one is absolute minimum	If $f(x)$ given (i) Find $f'(x)$ (ii) Find $f''(x)$ (iii) Solve $f''(x) = 0$ and get values of x. (iv) Fix the limits on both sides of x values (v) Check the sign of $f''(x)$ in above limits If $f''(x) > 0$ concave upward If $f''(x) < 0$ concave downward

WORKING RULES

LOCAL MAXIMUM AND MINIMUM		POINT OF INFLECTION
FIRST DERIVATIVE	SECOND DERIVATIVE	
If $f(x)$ given (i) Find $f'(x)$ (ii) Solve $f'(x) = 0$ and get C.N (iii) Fix the limits on both sides of C.N (iv) Check the sign of $f'(x)$ in above limits If $f'(x) > 0$ Increasing If $f'(x) < 0$ Decreasing If the results is : + ve to – ve to local maximum – ve to + ve to local minimum	If $f(x)$ given (i) Find $f'(x)$ (ii) Solve $f'(x) = 0$ and get C.N (iii) Find $f''(x)$ (iv) Check the sign of $f'(x)$ in critical number If the sign of If $f''(x) > 0$ local minimum If $f''(x) < 0$ local maximum $\int_0^\infty x^n e^{-ax} dx = \frac{\lfloor n}{a^{n+1}}$	If $f'(x)$ given (i) Find $f'(x)$ (ii) Find $f''(x)$ (iii) Solve $f''(x) = 0$ and get values of x. (iv) Fix the limits on both sides of x values (v) Check the sign of $f''(x)$ in above limits If $f''(x) > 0$ concave upward If $f''(x) < 0$ concave downward If a point has on both sides at the point the curve has a point of inflection

Notes:

CHAPTER 8

DIFFERENTIAL & PARTIAL DERIVATIVES

Partial Differentiation:

Second order derivatives defined by

$$\frac{\partial^2 F}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial x}\right) = Fxx$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y}\right) = Fxy$$

$$\frac{\partial^2 F}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x}\right) = Fyx$$

$$\frac{\partial^2 F}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial y}\right) = Fyy$$

✶ Suppose ω is a function of two variables x and y are functions of a single variable 't' then

$$\frac{d\omega}{dt} = \frac{\partial \omega}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial \omega}{\partial y}\cdot\frac{dy}{dt}$$

✶ Suppose ω is a function of two variables x and y are functions of a two variables s and t then

$$\frac{\partial \omega}{\partial s} = \frac{\partial \omega}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial \omega}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial \omega}{\partial t} = \frac{\partial \omega}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial \omega}{\partial y}\cdot\frac{\partial y}{\partial t}$$

Linear approximation:

Let $A = \{(x, y) \,/\, a < x < b,\ c < y < d\} < \mathrm{R}^2$

$F : A \to R$ and $(x_0, y_0) \in A$

(i) The linear approximation of F at $(x_0, y_0) \in A$ is defined to be

$$F(x, y) = F(x_0, y_0) + \left.\frac{\partial F}{\partial x}\right|_{(x_0, y_0)} (x_0, y_0) + \left.\frac{\partial F}{\partial x}\right|_{(x_0, y_0)} (y - y_0)$$

(ii) The differential of F is defined to be

$$dF = \frac{\partial F}{\partial x}(x, y)\,dx + \frac{\partial F}{\partial y}(x, y)\,dy$$

where $dx = \Delta_x$ and $dy = \Delta_y$

Euler's Theorem:

If $f(x, y)$ is a homogeneous function of degree n, then

$$x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} = nf$$

PROPERTIES OF DIFFERENTIALS:

Here we consider real-valued functions of real variable.

(1) If f is a constant function , then $df = 0$

(2) If $f(x) = x$ identity function, then $df = 1\,dx$

(3) If f is differentiable and $c \in \mathbb{R}$, then $d(cf) = c\,f'(x)\,dx$

(4) If f, g are differentiable, then, $d(f + g) = df + dg = f'(x)\,dx + g'(x)\,dx$

(5) If f, g are differentiable, then, $d(fg) = fdg + gdf = (f(x)\,g'(x) + f'(x)g(x))\,dx$

(6) If f, g are differentiable, then,

$$d(f/g) = \frac{gdf - fdg}{g^2} = \frac{g(x)f'(x) - f(x)g'(x)}{g^2 x}dx,$$

where $g(x) \neq 0$

(7) If f, g are differentiable and $h = f \circ g$ is defined, then, $dh = f'(g(x))\,g'(x)\,dx$.

(8) If $h(x) = e^{f(x)}$, then $dh = e^{f(x)} f'(x)\,dx$

(9) If $f(x) > 0$ for all x and $g(x) = \log(f(x))$, then

$$dg = \frac{f'(x)}{f(x)}dx$$

Absolute error = actual value – approximate value

$$\text{Relative error} = \frac{\text{absolute error}}{\text{actual error}}$$

Percentage error = relative error × 100 (or)

$$\frac{\text{absolute error}}{\text{actual error}} \times 100$$

Euler's Theorem: $\Rightarrow x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = nu$

CHAPTER 9

APPLICATIONS OF INTEGERALS

BASIC INTEGRATION FORMULAS

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$ (power rule)
2. $\int dx = x + c$
3. $\int e^x\,dx = e^x + c$
4. $\int \frac{1}{x}\,dx = \log|x| + c$
5. $\int \sin x\,dx = -\cos x + c$
6. $\int \cos x\,dx = \sin x + c$

Notes:

7. $\int \sec^2 x\, dx = \tan x + c$

8. $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

9. $\int \sec x \tan x\, dx = \sec x + c$

10. $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

11. $\int a^x dx = \dfrac{a^x}{\log a} + c$

12. $\int \dfrac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

13. $\int \dfrac{1}{1+x^2} dx = \tan^{-1} x + c$

14. $\int \tan x\, dx = -\log(\sec x) + c$

15. $\int \cot x\, dx = \log(\sin x) + c$

16. $\int \operatorname{cosec} x\, dx = \log(\operatorname{cosec} x - \cot x) + c$

17. $\int \sec x\, dx = \log(\sec x + \tan x) + c$

18. $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c$

19. $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + c$

20. $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right| + c$

21. $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right) + c$

22. $\int \dfrac{dx}{\sqrt{a^2+x^2}} = \log\left|x+\sqrt{a^2+x^2}\right| + c$

23. $\int \dfrac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + c$

24.

$$\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$$

25.

$$\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x+\sqrt{x^2-a^2}) + c$$

26. $\int \sqrt{a^2-x^2}\, dx = \dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\sin^{-1}\left(\dfrac{x}{a}\right) + c$

27. $\int \dfrac{1}{x^2+a^2} dx = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + c$

28. $\int \dfrac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c$

29. $\int \dfrac{1}{x} dx = \log x + c$

30. $\int \dfrac{1}{x\sqrt{x^2-a^2}} dx = \dfrac{1}{a}\sec^{-1}\left(\dfrac{x}{a}\right) + c$

IMPORTANT PROPERTIES OF INTEGRAL	
$\int_a^b f(x)\,dx = F(b) - F(a)$	$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$
$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$	$\int_a^b f(x)\,dx = \int_a^b f(a-x)\,dx$
$\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if $(2a-x) = f(x)$	
$\int_0^{2a} f(x)\,dx = 0$ if $(2a-x) = f(x)$	
$\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$ if f is even	
$\int_{-a}^a f(x)\,dx = 0$ if f is odd	

BERNOULLI'S FORMULA

$$\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_2 + \cdots$$

GAMMA FORMULAS

$$\underline{|(n)} = \int_0^\infty e^{-x}x^{n-1}dx = (n-1)!$$

Notes:

$$\int_0^{\infty} e^{-ax}x^n\,dx = \frac{n!}{a^{n+1}}$$

DEFINITE INTEGRAL AS THE LIMIT OF A SUM

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\frac{b-a}{n}\sum_{r=1}^{n} f\left(a+(b-a)\frac{r}{n}\right)$$

$$\int_0^1 f(x)\,dx = \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n} f\left(\frac{r}{n}\right) = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$$

REDUCTION FORMULAS

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx =$$

$$\begin{cases} \dfrac{(n-1)}{n}\times\dfrac{(n-3)}{(n-2)}\times\ldots\times\dfrac{1}{2}\times\dfrac{\pi}{2}, & \text{if } n = 2,4,6 \\ \dfrac{(n-1)}{n}\times\dfrac{(n-3)}{(n-2)}\times\ldots\times\dfrac{2}{3}, & \text{if } n = 3,5,7 \end{cases}$$

Volume of the solid of revolution

The volume of the solid of revolution about x-axis is

$$V = \pi\int_a^b y^2\,dx.$$

The volume of the solid of revolution about y-axis is

$$V = \pi\int_a^b x^2\,dy.$$

CHAPTER 10

ORDINARY DIFFERENTIAL EQUATIONS

Method of solving:

Type-I: Variable separable:

An equation of the form

$f_1(x)\,g_1(y)\,dx + f_2(x)\,g_2(y)\,dy = 0$

The equation can be written as $\dfrac{f_1(x)}{f_2(x)}\,dx = -\dfrac{g_2(y)}{g_1(y)}\,dy$

Type-II: Homogeneous equations:

put $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ and

use variable separable method.

Type-III: First order Linear Differential Equation:

(i) A first order linear differential equation of the form

$\dfrac{dy}{dx} + Py = Q$ where P and Q are function of x only.

$$ye^{\int Pdx} = \int Qe^{\int Pdx}\,dx + c.$$

(ii) Form of equation: $\dfrac{dx}{dy} + Px = Q$

where P and Q are function of y only/

$$xe^{\int Pdy} = \int Qe^{\int Pdy}\,dy + c$$

where $e^{\int Pdx}$ and $e^{\int Pdy}$ are known as the integrating factor (I.F.)

1. A differential equation is any equation which contains at least one derivative of an unknown function , either ordinary derivative or partial derivative.
2. The order of a differential equation is the highest derivative present in the differential equation.
3. If a differential equation is expressible in a polynomial form, then the integral power of the highest order derivative appears is called the degree of the differential equation.
4. If a differential equation is not expressible to polynomial equation form having the highest order derivative as the leading term then that the degree of the differential equation is not defined.
5. If a differential equation contains only ordinary derivatives of one or more functions with respect to a single independent variable , it is said to be an ordinary differential equation (ODE).
6. An equation involving only partial derivatives of one or more function of two or more independent variables is called a partial differential equation (PDE).
7. The result of eliminating one arbitrary constant yields a first order differential equation and that of eliminating two arbitrary constants leads to a second order differential equation and so on.
8. A solution of a differential equation is an expression for the dependent variable in terms of the independent variable(s) which satisfies the differential equation.
9. The solution which contains as many arbitrary constants as the order of the differential equation is called the general solution.

Notes:

10. If we given particular values to the arbitrary constants in the general solution of differential equation, the resulting solution is called a Particular solution.
    The order of a differential equation is the order of the highest order derivative occurring in it.
    The degree of the differential equation is the degree of the highest order derivative which occur in it.

$$e^{\log A} = A$$

$$e^{m \log A} = A^m$$

$$e^{-m \log A} = \frac{1}{A^m}$$

LINEAR DIFFERENTIAL EQUATION

If linear equation is in the form of $\frac{dy}{dx} + py = Q$,

then solution of $y\, e^{\int pdx} = \int Q\, e^{\int pdx} dx + c$

If linear equation is in the form of $\frac{dx}{dy} + px = Q$,

then solution of $x\, e^{\int pdy} = \int Q\, e^{\int pdy} dy + c$

CHAPTER 11

PROBABILITY DISTRIBUTIONS

PROBABILITY DENSITY FUNCTION (pdf):

$$P(a \le x \le b) = \int_a^b f(x)\, dx$$

It is non-negative for all real X.

$$\boxed{\int_{-\infty}^{\infty} f(x)\, dx = 1}$$

Mathematical Expectation (Mean):

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx \text{ (or)}$$

$$\sum_{i=1}^{n} Pi\ xi = P_1 x_1 + P_2 x_2 + \ldots + P_i x_i$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx \text{ (or)}$$

$$\sum_{i=1}^{n} Pi\ xi^2 = P_1 x_1^2 + P_2 x_2^2 + \ldots + P_i x_i^2$$

VARIANCE (X):

$\mathrm{Var}(X) = E(X^2) - [E(x)]^2$

PROPERTIES:

1. $E(c) = c$ where c is a constant
2. $E(cX) = c\,E(x)$
3. $E(aX + b) = a\,E(x) + b$
4. $\mathrm{Var}(X + c) = \mathrm{var}(X)$
5. $\mathrm{Var}(aX) = a^2\,\mathrm{Var}(X)$
6. $\mathrm{Var}(C) = 0$ Variance of a constant is zero.

PROBABILITY MASS FUNCTION

The mathematical definition of discrete probability function $p(x)$ is a function that satisfies the following properties

1. The probability that X can take a specific values x is $p(x)$, i.e., $P(X = x) = p(x) = p_x$
2. $p(x)$ is non-negative for all real x.
3. The sum of $p(x)$ over all possible values of X is one. That is $pi = 1$ where j represents all possible values that X an have and pi is the probability at $X = xi$

If $a_1, a_2, \ldots, a_m, a, b_1, b_2, \ldots b_n, b$ be the values of the discrete random variable X in ascending order then

(i) $P(X \ge a) = 1 - P(X < a)$

(ii) $P(X \le a) = 1 - P(X > a)$

(iii) $P(a \le X \le b) = P(X = a) + P(x = b_1) + P(X = b_2) + \ldots + P(X = b_n) + P(X = b)$

PROPERTIES OF CUMULATIVE DISTRIBUTION FUNCTION :

1. $F(x)$ is a non-decreasing function of x.
2. $0 \le F(x) \le 1, -\infty < x < \infty$
3. $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$
4. $F(\infty) = \lim_{x \to \infty} F(x) = 1$
5. $F(X = X_n) = F(X_n) - F(X_{n-1})$

PROPERTIES OF DISTRIBUTION FUNCTION:

1. $F(x)$ is a non-decreasing function of x.
2. $0 \le F(x) \le 1, -\infty < x < \infty$

Notes:

3. $F(-\infty) = \lim\limits_{x\to-\infty}\int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{-\infty} f(x)\,dx = 0$

4. $F(\infty) = \lim\limits_{x\to\infty}\int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{\infty} f(x)\,dx = 0$

5. For all real constant a and b, $a \le b$, $P(a \le x \le b) = F(b) - F(a)$

6. $f(x) = \dfrac{d}{dx}F(x) = i.e., F'(x) = f(x)$

MATHEMATICAL EXPECTATION:

$$E(X) = p_1x_1 + p_2x_2 + \cdots + p_nx_n = \sum_{i=1}^{n} p_ix_i$$

$$\sum_{i=1}^{n} p_i = 1$$

THEORETICAL DISTRIBUTION:

BINOMIAL FUNCTION:

$$P(X = x) = p(x) = \begin{cases} nc_x,\ p^x q^{n-x}, & x = 0, 1, 2, \ldots, n \\ 0 & elsewhere \end{cases}$$

MEAN: np

VARIANCE: npq

STANDARD DEVIATION: $\sqrt{npq}$

POISSON FUNCTION:

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{\lfloor x},\ \ x = 0,\ 1,\ 2, \text{ for some } \lambda > 0$$

MEAN: λ

VARIANCE: λ

STANDARD DEVIATION: $\sqrt{\lambda}$

NORMAL FUNCTION: $\dfrac{1}{\sigma\sqrt{2\pi}}e^{\frac{1}{-2}\left(\frac{x-\mu}{\sigma}\right)^2};\ -\infty < x < \infty$

MEAN: μ

VARIANCE: σ^2

STANDARD DEVIATION: σ

CHAPTER 12

DISCREATE MATHEMATICS

Complement laws:

(i) $p \vee \neg p \equiv T$ and $p \wedge \neg p \equiv F$

(ii) $\neg T \equiv F$ and $\neg F \equiv T$

Involution law or double negation law:

$\neg(\neg p) = p$

De Morgan's law:

(i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(ii) $\neg(p \vee q) \equiv \neg \text{p} \wedge \neg q$

TRUTH TABLE

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

A non-empty set G, together with an operation * i.e., (G, *) is said to be a group if it satisfies the following axioms

1. Closure Axiom: $a, b \in G \Rightarrow a * b \in G$
2. Associative Axiom: $\forall\ a, b, c \in G, (a * b) * c = a * (b * c)$
3. Identity Axiom: There exists an element $e \in G$ such that $a * e = e * a = a, \forall\ a \in G$
4. Inverse Axiom: $\forall\ a \in G$ there exists an element $a^{-1} \in G$ such that $a^{-1} * a = a * a^{-1} = e$
   e is called the identity element of G and a^{-1} is called the inverse of a in G
5. Commutative Axiom: $a, b \in G \Longrightarrow a * b = b * a$
   A group G satisfies the commutative axiom then the group G is called an Abelian group.

Idempotent laws:

(i) $p \vee p = p$ (ii) $p \wedge p = p$

Commutative laws:

(i) $p \vee q = q \vee p$

(ii) $p \wedge q = q \wedge p$

Associative laws:

(i) $p \vee (q \vee r) = (p \vee q) \vee r$

(ii) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

Distributive laws:

(i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

Identity laws:

(i) $p \vee T = T$ and $p \vee F = p$

(ii) $p \wedge T = P$ and $p \wedge F = F$

Absorption laws:

(i) $p \vee (p \wedge q) = p$ (ii) $p \wedge (p \vee q) \equiv p$

ALGEBRIC IDENTITIES

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $a^2 - b^2 = (a + b)(a - b)$

Notes:

4. $a^2 + b^2 = (a+b)^2 - 2ab$ (or) $(a-b)^2 + 2ab$
5. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
6. $(a+b)^3 = a^3 + b^3 + 3ab\,(a+b)$
7. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
8. $(a-b)^3 = a^3 - b^3 - 3ab\,(a-b)$
9. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
10. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
11. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

IMPORTANT TRIGONOMETRY IDENTITIES

I	$90^\circ - \theta$ $360^\circ + \theta$	$\mathbf{A}\rightarrow \left.\begin{matrix}\sin\theta\\ \cos\theta\\ \tan\theta\\ \sec\theta\\ \operatorname{cosec}\theta\\ \cot\theta\end{matrix}\right\}+\text{ve}$
II	$90^\circ + \theta$ $180^\circ - \theta$	$\mathbf{S}\rightarrow \left.\begin{matrix}\sin\theta\\ \operatorname{cosec}\theta\end{matrix}\right\}+\text{ve}$ $\left.\begin{matrix}\cos\theta\\ \tan\theta\\ \sec\theta\\ \cot\theta\end{matrix}\right\}-\text{ve}$
III	$180^\circ + \theta$ $270^\circ - \theta$	$\mathbf{T}\rightarrow \left.\begin{matrix}\tan\theta\\ \cot\theta\end{matrix}\right\}+\text{ve}$ $\left.\begin{matrix}\sin\theta\\ \cos\theta\\ \operatorname{cosec}\theta\\ \sec\theta\end{matrix}\right\}-\text{ve}$
IV	$270^\circ + \theta$ $360^\circ - \theta$	$\mathbf{C}\rightarrow \left.\begin{matrix}\cos\theta\\ \sec\theta\end{matrix}\right\}+\text{ve}$ $\left.\begin{matrix}\sin\theta\\ \operatorname{cosec}\theta\\ \tan\theta\\ \cot\theta\end{matrix}\right\}-\text{ve}$

RECIPROCALS IDENTITIES

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

QUOTIENT ANGLES

$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan x = \frac{\cos x}{\sin x}$$

PYTHAGOREAN IDENTITIES

$$\sin^2 x + \cos^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\operatorname{cosec}^2 x = 1 + \cot^2 x$$

DOUBLE IDENTITIES

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\quad = 2\cos^2 x - 1$

$\quad = 1 - 2\sin^2 x$

$$\tan^2 x = \frac{2\tan x}{1 - \tan^2 x}$$

ADDITION AND SUBTRACTION

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

****

Notes:

PHYSICS
CHAPTER 1
ELECTROSTATICS

- Electrostatic force between charges q_1 and q_2,

  $$F = \vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{21}$$

- Value of $k = \frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\,\mathrm{NM}^2C^{-2}$
- Value of $\varepsilon = 8.8544\times10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}$
- Total charge $q = n\times e$
  Number of electrons × Charge of an electron
- Components of force F
  $F_1 = F\cos\theta;\ F_2 = F\sin\theta;\ |F| = \sqrt{F_1^2 + F_2^2}$
- Relative permittivity or Dielectric constant $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$
- Force between charges in medium $F_m = \frac{F_{\text{air}}}{\varepsilon_r}$
- Electrostatic field, $E = \frac{\text{force}}{\text{charge}} = \frac{F}{q} \Rightarrow F = qE$
- Electric field due to a point charge, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
- Electric dipole moment, $\vec{p} = q\times 2a\hat{i}$
- Electric field due to a dipole at a point on the axial line,

  $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}(r >> a)$$

- Electric field due to a dipole at a point on the equatorial line,

  $$E = \vec{E}_0 = \frac{-1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}(r >> a)$$

- Magnitude of torque $\tau = \vec{p}\times\vec{E} = pE\sin\theta\ (p = q2a)$
- Electric potential at a point due to a point charge,

  $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

- Electric potential energy of dipole
  $U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$
- Electric potential at a point due to an electric dipole

  $$V = \frac{p}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}$$

- Electric flux $= \frac{q}{\varepsilon_0} \Rightarrow \phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$
- Electric field due to infinite long straight charged wire,

  $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

- Electric field due to a plane sheet of charge,

  $$E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{A}\frac{1}{2\varepsilon_0}$$

- Electric field at a point between two parallel sheets of charge,

  $$E = \frac{\sigma}{\varepsilon_0}$$

- Electric field due to a uniformly charged sphere -

  at a point on the surface of the sphere, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{r}$

  at a point outside the sphere, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

  at a point inside the sphere $E = 0$
- Capacitance of a conductor $C = \frac{q}{v}$
- Work done by a charge $W = qV$
- Charge density, $\sigma = \frac{q}{A}$
- Capacitance of a parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$
- With a dielectric slab, $C = \frac{\varepsilon_0 A}{\left[(d-t)+\frac{t}{\varepsilon_r}\right]}$
- With the dielectric completely filled the capacitor

  $$C^1 = \frac{\varepsilon_0\varepsilon_r A}{d} = C\times\varepsilon_r$$

- Energy stored in a capacitor, $E = \frac{1}{2}CV^2$
- Capacitance of a spherical capacitor,

Notes:

$$C = 4\pi\varepsilon_0 A \text{ or } C = \frac{A}{9\times10^{9}}$$

- Equivalent capacitance

  C_1 and C_2 in series, $C_S = \frac{C_1C_2}{C_1+C_2}$; $\frac{1}{C_S} = \frac{1}{C_1}+\frac{1}{C_2}$

  C_1 and C_2 in parallel $C_p = C_1 + C_2$
- Polarisation, $\vec{p} = \chi_e \vec{E}_{\text{ext}}$ (x_e – electric susceptibility)

CHAPTER 2

CURRENT ELECTRICITY

- The current, I flowing in a conductor $I = \frac{dQ}{dt}$

  where dQ is the charge that flows dt through a cross-section in a time interval dt.
- SI unit of current is ampere (A). 1A = 1 C s^{-1}.
- The current density J in a conductor is the current flowing per unit area,

  $\left(J = \frac{I}{A}\right)$
- Current is a scalar but current density is a vector.
- The general form of Ohm's law $\vec{J} = \sigma\vec{E}$
- Practical form of Ohm's law states that $V \propto I$, or $V = IR$ where I is the current.
- The resistance R of a conductor is $R = \frac{V}{I}$.

  SI unit of resistance is ohm (Ω) and $1\Omega = \frac{1V}{1A}$
- The resistance of a material $R = \rho\frac{l}{A}$ where l is length of the material and A is the area of cross section.
- The resistivity of a material determines how much resistance it offers to the flow of current.
- The equivalent resistance (R_s) of several resistances (R_1, R_2, R_3........) connected in series combination is R_s = (R_1, R_2, R_3........).
- The equivalent resistance (R_p) of several resistances (R_1, R_2, R_3........) connected in parallel combination is

  $$\frac{1}{R_P} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+.....$$
- Kirchoff's first rule (Current rule or junction rule): The algebraic sum of the currents at any junction is zero.
- Kirchoff's second rule (Voltage rule or loop rule): In a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit.
- Electric power is the rate at which energy is transformed.
- If a current I flows across a potential difference V, the power delivered to the circuit is P = IV.
- In a resistor R, the electrical power converted to heat is

  $$P = I^2R = \frac{V^2}{R}$$
- The energy equivalent of one kilowatt-hour (kWh) is 1 kWh = 3.6 × 10^6 J.
- Metre bridge is one form of Wheatstone's bridge.
- Potentiometer is used to compare potential differences.
- Joule's law of heating is $H = Vlt$ (or) $H = I^2Rt$.
- Thermoelectric effect: Conversion of temperature differences into electrical voltage and vice versa.

CHAPTER 3

MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT

- A vertical plane passing through geographic axis is called geographic meridian.
- A vertical plane passing through magnetic axis is called magnetic meridian.
- The angle between magnetic meridian at a point with the geographic meridian is called the declination or magnetic declination.
- The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination at that point.
- The magnetic moment is defined as the product of its pole strength and magnetic length. It is a vector quantity, denoted by $\vec{P}_m$.
- The region surrounding magnet where magnetic pole of strength unity experiences a force is known as magnetic

Notes:

field. It is a vector quantity and denoted by $\vec{B}$. Its unit is N A^{-1} m^{-1}.

- ♦ The number of magnetic field lines crossing normally through a given area is called magnetic flux Φ_B. It is a scalar quantity. In SI unit, magnetic flux Φ_B is weber, symbol Wb.
- ♦ Statement of Coulomb's law in magnetism "The force of attraction or repulsion between two magnetic poles is proportional to the product of their pole strengths and inversely proportional to the square of distance between them".
- ♦ Magnetic dipole kept in a uniform magnetic field experiences torque.
- ♦ Tangent galvanometer is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.
- ♦ Tangent law is $B = B_H \tan \theta$.
- ♦ The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. It is a vector quantity and denoted by $\vec{H}$ and its unit is A m^{-1}.
- ♦ The measure of ability of the material to allow the passage of magnetic lines of force through it is known as magnetic permeability.
- ♦ The net magnetic moment per unit volume of material is known as intensity of magnetisation.
- ♦ Magnetic susceptibility is defined as the ratio of the intensity of magnetisation $(\vec{I})$ induced in the material due to the magnetising field $(\vec{H})$.
- ♦ Magnetic materials are classified into three categories: diamagnetic, paramagnetic and ferromagnetic materials.
- ♦ The lagging of magnetic induction $\vec{B}$ behind the cyclic variation in magnetising field $\vec{H}$ is defined as "Hysteresis", which means "lagging behind".
- ♦ The right hand thumb rule "Assume that we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the rest of the fingers encircling the wire point in the direction of the magnetic field lines produced".
- ♦ Maxwell right hand cork screw rule "If we rotate a screw by a screw driver, then the direction of current is same as the direction in which screw advances, and the direction of rotation of the screw will determine the direction of the magnetic field".
- ♦ Ampère's circuital law is $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$
- ♦ Magnetic field inside the solenoid is $B = \mu_0 nI$, where n is the number of turns per unit length.
- ♦ Magnetic field interior to the toroid is $B = \mu_0 nI$, where n is the number of turns per unit length.
- ♦ Lorentz force is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.
- ♦ Charged particle moving in a uniform magnetic field will undergo circular motion.
- ♦ Fleming's Left Hand Rule: Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If we keep the forefinger in the direction of magnetic field, the middle finger in the direction of the electric current, then the thumb points in the direction the force experienced by the conductors.
- ♦ One ampere is defined as that constant current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of 2×10^{-7} newton per metre length of the conductor.
- ♦ When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but net torque is not zero. The magnitude of net torque is τ = NABI sin θ.
- ♦ Moving coil galvanometer is an instrument used for the detection and measurement of small currents.
- ♦ In moving coil galvanometer, current passing through the galvanometer is directly proportional to the deflection. Mathematically, $I = G\theta$, where $G = \frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.
- ♦ Current sensitivity is defined as the deflection produced per unit current flowing through it,

$$I_s = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_s = \frac{1}{G}.$$

- ♦ Voltage sensitivity is defined as the deflection produced per unit voltage which is applied across it,

$$V_s = \frac{\theta}{V} = \frac{1}{GR_g} = \frac{I_S}{R_g}$$

where, R_g is the resistance of galvanometer.

Notes:

- Ammeter is an instrument used to measure current in an electrical circuit.
- A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer.
- An ideal ammeter has zero resistance.
- Voltmeter is an instrument used to measure potential difference across any element in an electrical circuit.
- A galvanometer can be converted into suitable voltmeter of given range by connecting a suitable high resistance Rh in series with the given galvanometer.
- An ideal voltmeter has infinite resistance.

CHAPTER 4

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

- Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit. This phenomenon is known as electromagnetic induction.
- Faraday's first law states that whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.
- Faraday's second law states that the magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.
- Lenz's law states that the direction of the induced current is such that it always opposes the cause responsible for its production.
- Lenz's law is established on the basis of the law of conservation of energy.
- Fleming's right hand rule states that if the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.
- Even for a conductor in the form of a sheet or a plate, an emf is induced when magnetic flux linked with it changes. The induced currents flow in concentric circular paths called Eddy currents or Foucault currents.
- Inductor is a device used to store energy in a magnetic field when an electric current flows through it.
- If the flux linked with the coil is changed, an emf is induced in that same coil. This phenomenon is known as self-induction. The emf induced is called self-induced emf.
- When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction and the emf is called mutually induced emf.
- AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy.
- In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called three-phase AC generators.
- Transformer is a stationary device used to transform AC electric power from one circuit to another without changing its frequency.
- The efficiency of a transformer is defined as the ratio of the useful output power to the input power.
- An alternating voltage is a voltage which changes polarity at regular intervals of time and the resulting alternating current changes direction accordingly.
- The average value of alternating current is defined as the average of all values of current over a positive half-circle or negative half-circle.
- The root mean square value or effective value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle.
- A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity. Such a rotating vector is called a phasor.
- When the frequency of the applied alternating source is equal to the natural frequency of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in electrical resonance.
- The magnification of voltages at series resonance is termed as Q–factor.
- Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It depends on the components of the circuit.
- Whenever energy is given to a LC circuit, the electrical oscillations of definite frequency are generated. These oscillations are called LC oscillations.
- During LC oscillations, the total energy remains constant. It means that LC oscillations take place in accordance with the law of conservation of energy.

Notes:

CHAPTER 5

ELECTROMAGNETIC WAVES

- Velocity of EM wave in vacuum $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

  μ - permeability of free space; ε_0 - permittivity of free space
- Frequency of oscillation of charges between plates

  $v = \frac{1}{2\pi\sqrt{LC}}$

  C = Capacitance, L - inductance (small wire)
- The speed of electromagnetic wave, $v = \frac{E_0}{B_0}$

  E_0 = Amplitude of oscillating electric field;

  B_0 = Amplitude of oscillating magnetic field;

Intensity $(I) \frac{\text{Power } (P)}{\text{Surface Area } (A)} = \frac{\text{Total electromagnetic energy}}{\text{Surface Area } (A) \times \text{time } (t)}$

- The average energy density of electromagnetic wave is

  $u = \varepsilon_0 E^2 = \frac{1}{\mu_0} \times B^2$

  μ_0 = permeability of free space; ε_0 - permittivity of free space
- Momentum Imparted by electromagnetic wave on the surface is

  $p = \frac{U}{c} = \frac{\text{Energy}}{\text{Velocity of light}}$
- Ampere's Maxwell law:

  $\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$
- Faraday's Law: $\oint \vec{E} \cdot \vec{dl} = \frac{d}{dt} \Phi_B ; \Phi_B$ - Magnetic flux
- Gauss's Law: $\oint \vec{E} \cdot \vec{dA} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$
- Ampere's circuital Law:

  $\oint \vec{B} \cdot \vec{dl} = \mu_0 I_0 ; I_c$ conduction current
- Refractive Index of the medium, $\mu = \sqrt{\varepsilon_r \ \mu_r}$

CHAPTER 6

RAY OPTICS

- A ray of light gives the direction of light.
- Law of reflection is, $i = r$
- Paraxial rays are the rays travelling close to the principal axis of the mirror and make small angles with it.
- The relation between focal length and radius of curvature in spherical mirror is,

  $2f = R$ (or) $f = \frac{R}{2}$
- Cartesian sign conventions are to be followed to trace image formed by spherical mirrors.
- The mirror equation is, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
- The magnification in spherical mirror is,

  $m = \frac{h'}{h} = -\frac{v}{u}, \ m = \frac{h'}{h} = \frac{f-v}{f} = \frac{f}{f-u}$
- Light travels with lesser velocity in optically denser medium.
- Refractive index is the ratio of speed of light in vacuum to speed of light in medium, $n = \frac{c}{v}$
- Optical path is the equivalent path travelled in vacuum in the same time light travels through a optically denser medium.

  $d' = nd$
- Law of refraction also called as Snell's law in ratio form is,

  $\frac{\sin i}{\sin r} = \frac{n_2}{n_1}.$
- In product form is, $n_1 \sin i = n_2 \sin r$
- The relative refractive index of second medium with respect to first medium is, $n_{21} = \frac{n_2}{n_1}$
- The apparent depth is always lesser than actual depth. The equation for apparent depth is, $d' = \frac{d}{n}$
- The critical angle of incidence ic for a ray incident from a denser to rarer medium, is that angle for which the angle of refraction is 90°. For $i > i_c$, total internal reflection occurs.
- Equations for critical angle incidence is,

Notes:

$$\sin i_c = \frac{1}{n} \text{ (or) } i_c = \sin^{-1}\left(\frac{1}{n}\right)$$

- Snell's window is the restricted area of circular illumination which appears when seen from water due to critical angle incidence. The radius of the circular illumination is,

$$R = d\left(\frac{1}{\sqrt{n^2-1}}\right) \text{ (or) } R = \frac{d}{\sqrt{n^2-1}}$$

- Optical fibre makes use of critical angle incidence. The acceptance angle in optical fibre is, Here, n_1, n_2, n_3 are the refractive indices of core, cladding and surrounding medium respectively.
- Glass slabs produce a lateral displacement on the light falling on it. The equation for lateral shift is,

$$L = t\left(\frac{\sin(i-r)}{\cos(r)}\right)$$

- The equation for single spherical surface is,

$$\frac{n}{v} - \frac{1}{u} = \frac{(n-1)}{R}$$

- The focal length of the thin lens is positive for a converging lens and negative for a diverging lens. It is not based on the position of the focal point.
- The lens makers formula is, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- The lens equation is, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
- The magnification produced by the lens is,

$$m = \frac{h'}{h} = \frac{v}{u},\ m = \frac{h'}{h} = \frac{f}{f+u} \text{ (or) } m = \frac{h'}{h} = \frac{f-v}{f}$$

- The power of a lens is a measure of the degree of convergence (or) divergence of light falling on it. Power and focal length are inverse to each other.

$$P = \frac{1}{f}$$

- The unit of power is diopter (D) when the focal length is taken in meter.
- The effective focal length of lenses in contact is,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

- A prism produces deviation on the incident ray.
- Angle of deviation depends on angle of prism, angle of incidence and refractive index of material of prism given by the equation, $d = i_1 + i_2 - A$
- At minimum deviation, $i_1 = i_2$, $r_1 + r_2$, and the ray inside the prism is parallel to the base of the prism.
- The refractive index of prism depends on angle of prism and angle of minimum deviation given by the equation,

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

- When white light travels through a medium, different colours travel with different speeds leading to dispersion of light. Red colour travels faster than violet colour in a medium. In vacuum all the colours travel with the same speed.
- The angle of deviation produced by the small angled prism is, $\delta = (n-1)A$
- The angular separation between the two extreme colours (violet and red) in the spectrum is called angular dispersion.

$$\delta_V - \delta_R = (n_V - n_R)A$$

- Dispersive power is the measure of ability of the medium to disperse white light.

$$\omega = \frac{(n_v - n_r)}{(n-1)}$$

- Rainbow is formed by dispersion of light by droplets of water.
- The scattering of light by particles of size less than wavelength of light is called Rayleigh scattering. The intensity of light produced by Rayleigh scattering is,

$$I \propto \frac{1}{\lambda^4}$$

- Non-Rayleigh scattering is by suspended dust particles whose size is greater than the wavelength of light. This scattering is independent of wavelength of light.

CHAPTER 7

WAVE OPTICS

- Light has wave as well as particle nature.
- A *wavefront* is the locus of points which are in the same state or phase of vibration. Huygen's principle states the method of propagation of wavefront.

Notes:

- Light propagating as a wavefront. Point source produces spherical wavefront and source at infinity produces plane wavefront.
- Laws of reflection and refraction are proved by Huygens' principle.
- The phenomenon of addition or superposition of two light waves which produces increase in intensity at some points and decrease in intensity at some other points is called *interference* of light.
- If I_1 and I_2 are the intensities of interfering light, then

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2};\ I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

- If interfering lights have equal intensities of I0 with phase difference ϕ, then

$$I \propto 4a^2 \cos^2(\phi/2)\ [\because I \propto A^2]$$

$I_{\max} = 4I_0$ when, $\phi = 0, \pm 2\pi, 4\pi ...$

$I_{\min} = 0$ when, $\phi = \pm\pi, \pm 3\pi, 5\pi ...$

- Two light sources are said to be coherent if they produce waves which have same phase or constant phase difference, same frequency or wavelength monochromatic), same waveform and preferably same amplitude.
- Coherent sources are obtained by wavefront division, intensity division and real and virtual images of light source.
- Young's double slit uses wavefront division to obtain coherent sources.
- In Young's double slit experiment, the position of nth bright fringe is

$$y_n = n\frac{\lambda D}{d}$$

- In Young's double slit experiment, the position of nth dark fringe is,

$$y_n = \frac{2(n-1)}{2}\frac{\lambda D}{d}$$

- In Young's double slit experiment, the equation for bandwidth is, $\beta = \frac{\lambda D}{d}$
- Interference with polychromatic (white) light produces coloured interference fringes.
- Thin films appear coloured due to interference of white light.
- Transmitted light are, $2\mu d = n\lambda;\ 2\mu d = (2n-1)\frac{\lambda}{2}$ respectively.
- In thins films the equations for constructive and destructive interference for reflected rays are,

$$2\mu d = (2n-1)\frac{\lambda}{2};\ 2\mu d = n\lambda \text{ respectively.}$$

- Diffraction is bending of waves around sharp edges into the geometrically shadowed region.
- A spherical wave front undergoes diffraction in Fresnel diffraction.
- A plane wavefront undergoes diffraction in Fraunhofer diffraction.
- The equation for *n*th minimum in single slit diffraction is, $a \sin\theta = n\lambda$ (n^{th} minimum)
- The equation for *n*th maximum in single slit diffraction is,

$$a \sin\theta = (2n+1)\frac{\lambda}{2}\ (n^{th} \text{ maximum})$$

- Fresnel's distance is the distance upto which ray optics is obeyed and beyond which ray optics is not obeyed but, wave optics becomes significant. $z = \frac{a^2}{2\lambda}$
- Diffraction can also happen in grating which has multiple slits of thickness comparable to wavelength of light used. $(a + b) \sin\theta = m\lambda;\ \sin\theta = N m \lambda$
- Wavelength of monochromatic light and also different colours of polychromatic light can be determined. Using diffraction grating and spectrometer.
- According to *Rayleigh's criterion*, for two point objects to be just resolved, the minimum distance between their diffraction images must be in such a way that the central maximum of one coincides with the first minimum of the other and vice versa
- Angular resolution, $\theta = \frac{1.22\lambda}{a}$
- Special resolution, $r_0 = \frac{1.22\lambda f}{a}$
- Resolution is measured by the smallest distance which could be seen clearly without the blur due to diffraction.
- The phenomenon of restricting the vibrations of light (electric or magnetic field vector) to a particular direction perpendicular to the direction of propagation of wave is called *polarization* of light.
- If the vibrations of a wave are present in only one direction

Notes:

in a plane perpendicular to the direction of propagation of wave is said to be *polarised* or *plane polarised light*.

- The plane containing the vibrations of the electric field vector is known as the *plane of vibration*.
- The plane perpendicular to the plane of vibration and containing the ray of light is known as the *plane of polarisation*.
- In *plane polarised* light the intensity varies from maximum to zero for every rotation of 90° of the analyser.
- If the intensity of light varies between maximum and minimum for every rotation of 90° of the analyser, the light is said to be partially polarised light
- In intensity of transmitted light through two cross polaroids is given by Malus' Law. $I = I_0 \cos^2 \theta$
- Nicol prism separates ordinary and extraordinary rays by double refraction.
- Light scattered by molecules at perpendicular direction to the incident light is found to be plane polarised.
- A single convex lens can act as a simple microscope when object is within the focal length.
- In near point focusing, the image is formed at $D = 25$ cm.
- The magnification in near point focusing is, $m = 1 + \frac{D}{f}$
- In normal focusing, the image is formed at infinity, the magnification in normal focusing is, $m = \frac{D}{f}$
- The resolving power of microscope is, $R_M = \frac{1}{d_{\min}} = \frac{2(NA)}{1.22\lambda}$
- The resolving power of telescope is, $R_T = \frac{1}{r_0} = \frac{a}{1.22\lambda f}$
- Magnification in near point focusing in compound is, $m = m_0 m_e = \left(\frac{L}{f_0}\right)\left(1 + \frac{D}{f_e}\right)$ microscope
- Magnification in near point focusing in compound microscope is, $m = m_0 m_e = \left(\frac{L}{f_0}\right)\left(\frac{D}{f_e}\right)$
- Magnification in astronomical telescope is, $m = \frac{f_0}{f_e}$

  and the tube length is, $L = f_o + f_e$
- The correction lens for nearsightedness/myopic eye should be a convex lens with focal length, $f = -x$. Where x is the near distance upto which it is claearly seen.
- The correction lens for farsightedness/myopic eye should be a concave lens with focal length,

  $$f = \frac{y \times 25\text{ cm}}{y - 25\text{ cm}}.$$

  Where, the 25 cm is the near distance.
- The farsightedness arising due to aging is called presbyopia.
- Astigmatism is the defect arising due to different curvatures along different planes in the eye lens.

CHAPTER 8

DUAL NATURE OF RADIATION AND MATTER

- Particle is a material object which is considered as a tiny concentration of matter (localized in space and time) whereas wave is a broad distribution of energy (not localized in space and time).
- The liberation of electrons from any surface of a substance is called electron emission.
- The minimum energy needed for an electron to escape from the metal surface is called work function of that metal.
- 1 *eV* is equal to 1.602×10^{-19} J.
- The emission of electrons by supplying thermal energy is known as thermionic emission.
- Electric field emission occurs when a very strong electric field is applied across the metal.
- The emission of electrons due to irradiation of light is called photoelectric emission.
- Secondary emission is the process in which electrons are emitted due to the bombardment of fast moving electrons.
- The photoelectric current (i.e. the number of electrons emitted per second) is directly proportional to the intensity of the incident light.
- Stopping potential is that the value of the negative (retarding) potential given to the collecting electrode A which is just sufficient to stop the most energetic photoelectrons emitted and make the photocurrent zero.
- The stopping potential is independent of intensity of the incident light.
- Maximum kinetic energy of the photoelectrons is independent of intensity of the incident light.

Notes:

- For a given surface, the emission of photoelectrons takes place only if the frequency of incident light is greater than a certain minimum frequency called the threshold frequency.
- According to Planck, a matter is composed of a large number of oscillating particles (atoms) which vibrate with different frequencies.
- According to Einstein, the energy in light is not spread out over wavefronts but is concentrated in small packets or energy quanta.
- The individual light quantum of definite energy and momentum is called photon.
- Light behaves as a wave during its propagation and behaves as a particle during its interaction with matter.
- Photo electric cell or photo cell is a device which converts light energy into electrical energy.
- According to de Broglie hypothesis, all material particles like electrons, protons, neutrons in motion possess wave nature.
- These waves associated with them are called de Broglie waves or matter waves.
- Wave nature of the electron is used in the construction of electron microscope.
- Louis de Broglie hypothesis of matter waves was experimentally confirmed by Clinton Davisson and Lester Germer in 1927
- Whenever fast moving electrons fall on the materials, a highly penetrating radiations, namely x-rays, are emitted.
- Continuous *x*-ray spectrum consists of radiations of all possible wavelengths with a certain minimum wavelength λ_0.
- Characteristic x-ray spectra show some narrow peaks at some well–defined wavelengths when the target is hit by fast electrons.

CHAPTER 9
ATOMIC AND NUCLEAR PHYSICS

- A device used to study the conduction of electricity through partical gases at low pressure is known as gas discharge tube
- Charge per unit mass is known as specific charge or normalized charge, and it is independent of gas used and also nature of electrodes used in the experiment.
- The minimum distance between alpha particle and centre of the nucleus just before it gets reflected back by 180° is defined as distance of closest approach r_0.
- The impact parameter is defined as the perpendicular distance between the centre of the gold nucleus and the direction of velocity vector of alpha particle when it is at a large distance.
- According to Bohr atom model, angular momentum is quantized.
- The radius of the orbit in Bohr atom model is $r_n = a_0 \frac{n^2}{Z}$
- The radius of first orbit hydrogen atom is

  $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.529\ \text{Å}$ and it is also known as Bohr radius
- The velocity of electron in n^th^ orbit is

  $$v_n = \frac{h}{2\pi m a_0}\frac{Z}{n}\,\text{ms}^{-1}$$
- The fine structure constant is $\alpha = \frac{1}{137}$

  which is a dimensionless constant
- The total energy of electron in the n^{th} orbit is

  $$E_n = \frac{me^4}{8\varepsilon_0^2 h^2}\frac{Z^2}{n^2} = -13.6\frac{Z^2}{n^2}eV.$$
- The energy required to excite an electron from the lower energy state to any higher energy state is known as excitation energy and corresponding potential supplied is known as excitation potential.
- The minimum energy required to remove an electron from an atom in the ground state is known as binding energy or ionization energy.
- The potential difference through which an electron should be accelerated to get ionization energy is known as ionization potential.
- The wavelength of spectral lines of Lyman series lies in ultra-violet region
- The wavelength of spectral lines of Balmer series lies in visible region while those of Paschen and Brackett series lie in infra-red region
- The nucleus of element *X* having atomic number *Z* and mass number *A* is represented by ${}^{A}_{Z}X$.
- The emprical relation connecting radius of nucleus ($Z > 10$) *R* and mass number *A* is given by $R = R_0 A^{1/3}$ where $R_0 = 1.2$ F.

Notes:

- The density of nucleus $\rho = 2.3 \times 10^{17}$ kg m^{-3}, and the nuclear matter in a highly compressed state.
- If M, m_p and m_n are masses of a nucleus $\left({}^{A}_{Z}X\right)$, proton and neutron respectively , then the mass defect is $\Delta m = (Zm_p + Nm_n) - M$
- The average binding energy of nucleus $B.E. = (Zm_p + Nm_n - M)\, c^2$
- The binding energy per nucleon is maximum for iron which is 8.8 MeV.
- Alpha decay: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$
- β^- decay: ${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e^- + \bar{v}$
- β^+ decay: ${}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + e^+ + v$
- Gamma emission: ${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$
- Law of radioactive decay: $N = N_0\, e^{-\lambda t}$
- In general, after n half lives, the number of nuclei left undecayed is $N = \left(\frac{1}{2}\right)^n N_0$
- The relation between half-life and decay constant is $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{\lambda}$
- Mean life = $\tau = \frac{1}{\lambda};\ T_{1/2} = \frac{0.6931}{\lambda} = 0.6931\tau.$
- If a heavier nucleus decays into lighter nuclei, it is called nuclear fission
- If two lighter nuclei fuse to form heavier nucleus, it is called nuclear fusion
- In nuclear reactors, the nuclear chain reaction is controlled. In stars, the energy generation is through nuclear fusion.

CHAPTER 10

ELECTRONICS AND COMMUNICATION

- A Energy bands in solids are used to classify them into metals, insulators, and semiconductors
- In a N-type semiconductor, electrons are the majority charge carriers and holes are the minority carriers.
- In P-type semiconductor, holes are the majority charge carriers and electrons are the minority charge carriers.
- The thin region near the junction which is free from charge carriers (free electrons and holes) is called depletion region.
- When a PN junction diode is forward biased, the width of the depletion region decreases and the diode conducts.
- A PN junction diode in reverse biased condition functions as a open switch as it does not conduct. The width of the depletion region increases in this case.
- A forward biased PN junction diode functions as a rectifier. The process in which alternating voltage or alternating current is converted into direct voltage or direct current is known as rectification.
- The half wave rectifier rectifies one half of the input signal and produces a pulsating DC output.
- Full wave rectifier rectifies both halves of the input signal.
- Zener breakdown happens in a heavily doped PN junction diode when a strong electric field is applied.
- Avalanche breakdown occurs in lightly doped junctions which have wide depletion layers. It is due to the breaking of covalent bonds by the thermally generated minority charge carriers.
- Zener diode is a heavily doped PN junction diode which works in the reverse biased condition.
- Light emitting diode is a forward biased semiconductor device that emits emits visible or invisible light when energized. The recombination of minority charge carriers with the majority charge carriers in the respective regions release energy in the form of photons.
- A PN junction diode which converts an optical signal into electric signal is called a photodiode.
- When a photon of sufficient energy strikes the diode, it creates an electron-hole pair. These electrons and holes are swept across the p-n junction by the electric field created by reverse voltage before recombination takes place and in turn generates photo current.
- A solar cell, also known as photovoltaic cell, works on the principle of photovoltaic effect. Accordingly, the p-n junction of the solar cell generates emf when solar radiation falls on it. .
- A bipolar junction transistor (BJT) is a semiconductor device and there are two types in that: NPN and PNP
- A BJT can be operated in three different configurations: Common base, common emitter, common collector.
- The forward current gain in common base configuration α gives the ratio of the collector current to emitter current.
- The forward current gain in common emitter configuration β gives the ratio of the collector current to the base current

Notes:

- The BJT connected in common emitter configuration can be used as an amplifier. There exists a phase reversal of 1800 between the input signal and the amplified output signal.
- A transistor amplifier combined with a tank circuit and positive feedback acts as an oscillator
- The logic gates are logical circuits providing output only for a combination of inputs.
- De Morgan's First theorem states that the complement of the sum of two inputs is equal to the product of its complements.
- The second theorem states that the complement of the product of two inputs is equal to the sum of its complements.
- The basic elements required for the transmission and reception of a signal through long distance communication using electromagnetic waves are transducer, amplifier, carrier signal, modulator, power amplifier, medium of transmission, transmitting and receiving antenna, demodulator, detector.
- For long-distance transmission, the baseband signal is modulated with the carrier wave.
- If the amplitude of the carrier signal is modified with the instantaneous amplitude of the baseband signal then it is called amplitude modulation.
- The frequency of the carrier signal is modified with the instantaneous amplitude of the baseband signal in frequency modulation.
- The instantaneous amplitude of the baseband signal modifies the phase of the carrier signal keeping the amplitude and frequency constant in phase modulation
- If the EM waves transmitted by the transmitter glide over the surface of the earth to reach the receiver, then the propagation of EM waves is called ground wave propagation.
- The mode of propagation in which electromagnetic waves radiated from an antenna, directed upwards at large angles gets reflected by the ionosphere back to earth is called sky wave propagation.
- The process of sending and receiving information signal through space is called space wave communication.
- The satellite communication is a mode of communication of signal between transmitter and receiver via satellite.
- Fiber-optic communication is a method of transmitting information by sending pulses of light through an optical fiber.
- Radar basically stands for Radio Detection and Ranging System. It is one of the important applications of communication systems for remote sensing.
- Mobile Communication is used to communicate with others in different locations without the use of any physical connection like wires or cables.

CHAPTER 11

RECENT DEVELOPMENTS IN PHYSICS

- Physics is the basic building block for Science, Engineering, Technology and Medicine. Nano science is the science of objects with typical sizes of 1–100 nm.
- Nano means one-billionth of a metre that is 10–9 m.
- Nanotechnology is a technology involving the design, production, characterization, and applications of nano structural materials.
- If the particle of a solid is of size less than 100 nm, it is said to be a 'nano solid'.
- When the particle size exceeds 100 nm, it forms a 'bulk solid'.
- Nano form of the material shows strikingly different properties when compared to its bulk counterpart.
- Quantum confinement effects and surface effects are the two important phenomena that govern nano properties.
- Nanoscience and technology is the interdisciplinary area covering its applications in various fields.
- Nano scale structures existed in nature long before scientists began studying them in laboratories.
- There are two ways of preparing the nanomaterials, top down and bottom up approaches.
- Nanotechnology applications cover various fields.
- The major concern with nano application is that the nanoparticles have the dimensions same as that of the biological molecules such as proteins.
- Nano particles can easily get absorbed onto the surface of living organisms and they might enter the tissues and fluids of the body.
- The adsorbing nature depends on the surface of the nanoparticle.
- It is possible to deliver a drug directly to a specific cell in the body by designing the surface of a nanoparticle.
- Nanoparticles of a few nanometers size may reach well inside biomolecules, which is not possible for larger nanoparticles.
- Nanoparticles can also cross cell membranes.

Notes:

- The inhaled nanoparticles reach the blood and that may also reach other sites such as the liver, heart or blood cells.
- Robotics is an integrated study of mechanical engineering, electronic engineering, computer engineering, and science.
- Robot is a mechanical device designed with electronic circuitry and programmed to perform a specific task.
- The robotic system mainly consists of sensors, power supplies, control systems, manipulators and necessary software.
- The key components of a robot are Power conversion unit, Actuators, Electric motors, Pneumatic Air Muscles, Muscle wires, Piezo Motors and Ultrasonic Motors, Sensors, and Robot locomotion.
- Six main types of industrial robots are Cartesian, SCARA, Cylindrical, Delta, Polar and Vertically articulated robot.
- Six-axis robots are ideal for Arc Welding, Spot Welding, Material Handling, Machine Tending.
- Five major fields of robotics: Human-robot interface, Mobility, Manipulation, Programming and Sensors.
- The aim of artificial intelligence is to bring in human like behavior in robots.
- Artificial intelligence works on face recognition, providing response to players' actions in computer games, taking decisions based on previous actions, regulating the traffic by analyzing the density of traffic on roads and translate words from one language to another.
- Materials used to make robots: aluminum and steel are the most common metals.
- Aluminum is a softer metal and is therefore easier to work with.
- Steel is several times stronger.
- Due to the inherent strength of metal, robot bodies can be made using sheet, bar, rod, channel, and other shapes.
- Robots have many advantages in various applications but also have several disadvantages.
- In outer space robots are used for exploring stars, planets etc., investigation of the mineralogy of the rocks and soils on Mars, analysis of elements found in rocks and soils.
- Household robots are used as vacuum cleaners, floor cleaners, gutter cleaners, lawn mowing, pool cleaning, and to open and close doors.
- Industrial Robots are used for welding, cutting, robotic water jet cutting, robotic laser cutting, lifting, sorting, bending, manufacturing, assembling, packing, transport, handling hazardous materials like nuclear waste, weaponry, laboratory research, mass production of consumer and industrial goods.
- Nano-robots are being developed to be in the blood stream to perform small surgical procedures, to fight against bacteria, repairing individual cell in the body.
- The development in medical field has been proportional to the evolution of physics.
- The recent medical technology includes virtual reality, precision medicine, health wearables, artificial organs, 3D printing, wireless brain sensors, robotic surgery, smart inhalers.
- Particle physics deals with fundamental particles of nature. Protons and neutrons are made of quarks.
- Cosmology is the branch that involves the origin and evolution of the universe.
- Accelerated mass emits gravitational waves which are very weak.
- Black holes are the strongest source of gravitational waves.

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Notes:

CHEMISTRY

UNITWISE – IMPORTANT POINTS

UNIT 1

METALLURGY

- **Calcination:** Calcination is the process in which the concentrated ore is strongly heated in the absence of air.

$$ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$$

- Flux + Gangue → Slag
- Metallurgy relate to the science and technology of metals.
- A naturally occurring substance obtained by mining which contains the metal in free state or in the form of compounds like oxides, sulphides etc... is called a **mineral**.
- Minerals that contains a high percentage of metal, from which it can be extracted conveniently and economically are called **ores**.
- The extraction of a metal of interest from its ore consists of the following metallurgical processes.
  - (i) concentration of the ore
  - (ii) extraction of crude metal
  - (iii) refining of crude metal
- The extraction of crude metals from the concentrated ores is carried out in two steps namely,
  - (i) conversion of the ore into oxides of the metal of interest
  - (ii) reduction of the metal oxides to elemental metals.
- The graphical representation of variation of the standard Gibbs free energy of reaction for the formation of various metal oxides with temperature is called **Ellingham diagram**
- Ellingham diagram helps us to select a suitable reducing agent and appropriate temperature range for reduction.
- Similar to thermodynamic principles, electrochemical principles also find applications in metallurgical process.
- If E^0 is positive then the ΔG is negative and the reduction is spontaneous and hence a redox reaction is planned in such a way that the e.m.f of the net redox reaction is positive. When a more reactive metal is added to the solution containing the relatively less reactive metal ions, the more reactive metal will go into the solution.
- Generally the metal extracted from its ore contains some impurities such as unreacted oxide ore, other metals, nonmetals etc... Removal of such impurities associated with the isolated crude metal is called **refining process**.

UNIT 2

p-BLOCK ELEMENTS-I

- Inert pair effect: In heavier post-transition metals, the outer '*s*' electrons (*ns*) have a tendency to remain inert and show reluctance to take part in the bonding, which is known as **inert pair effect**.
- McAfee Process:

$$2Al_2O_3 + 3C + 6Cl_2 \longrightarrow 4AlCl_3 + 3CO_2$$

- **Allotropes of carbon:**

(i) Graphite:

> Each sheet is a hexagonal

> sp^2 hybridised

> C–C bond length of 1.41Å

(ii) Diamond:

> very hard

> sp^3 hybridised

> C–C bond length of 1.54Å

(iii) Fullerenes: newly synthesised allotropes of carbon

> C_{32}, C_{50}, C_{60}, C_{70}, C_{76} etc.,

- The elements in which their last electron enters the '*p*' orbital, constitute the ***p*-block elements**.
- The *p*-block elements have a general electronic configuration of $\boldsymbol{ns^2, np^{1-6}}$. The elements of each group have similar outer shell electronic configuration and differ only in the value of *n* (**principal quantum number**).
- Generally on **descending a group** the **ionisation energy decreases** and hence the **metallic character increases**.
- The ionisation enthalpy of elements in successive groups is higher than the corresponding elements of the previous group as expected.
- As we move down the 13th group, the **electronegativity first decreases** from boron to aluminium and then **marginally increases**.
- In *p*-block elements, the first member of each group differs from the other elements of the corresponding group.
- Some elements exist in more than one crystalline or molecular forms in the same physical state. For example, carbon exists as diamond and graphite. This

Notes:

phenomenon is called **allotropy (allotropism).**

- **Borax** is a sodium salt of **tetraboric acid**. It is obtained from **colemanite ore** by boiling its solution with sodium carbonate.
- **Boric acid** can be extracted from **borax** and **colemanite**.
- **Boric acid** has a **two dimensional layered structure.**
- The name **alum** is given to the **double salt of potassium aluminium sulphate** [$K_2SO_4.Al_2(SO_4)_3.24.H_2O$].
- **Carbon** is found in the native form as **graphite**.
- **Silicon** occurs as **silica** (sand and quartz crystal). **Silicate minerals** and **clay** are other important sources for silicon.
- **Catenation** is an ability of an element to form chain of atoms
- Carbon nanotubes, another recently discovered allotropes, have graphite like tubes with fullerene ends.
- **Silicones** or **poly siloxanes** are organo silicon polymers with general empirical formula (R_2SiO). Because of their very high thermal stability they are called high – temperature polymers.
- The mineral which contains silicon and oxygen in tetrahedral $[SiO_4]^{4-}$ units linked together in different patterns are called **silicates**.
- Types of Silicates:
  - **Ortho silicates** (**Neso silicates**), **Pyro silicate** (or **Soro silicates**), **Cyclic silicates** (or **Ring silicates**)
  - **Inosilicates**: **Chain silicates** (or **pyroxenes**), **Double chain silicates** (or **amphiboles**):
  - Sheet or phyllo silicates
  - Three dimensional silicates (or tecto silicates)
- Zeolites are three-dimensional crystalline solids containing aluminium, silicon, and oxygen in their regular three dimensional framework.
- Zeolites act as a molecular sieve for the removal of permanent hardness of water

UNIT 3

p-BLOCK ELEMENTS-II

- **Holmes signal:**

  Phosphine is used for producing smoke screen as it gives large smoke. In a ship, a pierced container with a mixture of calcium carbide and calcium phosphide liberates phosphine and acetylene when thrown into sea. The liberate phosphine catches fire and ignites acetylene. These burning gases serves as signal to the approaching ships. This is known as **Holmes signal**.
- **Phosphine:** Pyramidal shape.

  Phosphorous Pentachloride is a chlorinating agent.
- **Chlorine:** It is a greenish yellow gas with a pungent irritating odour.
- **Thermal stability:** HF > HI > HBr > HCl
- **Noble gases:** Noble gases are monoatomic, odourless, colourless, tasteless and non-inflammable.
- **Occurrence:** About 78 % of earth atmosphere contains dinitorgen (N_2) gas. It is also present in earth crust as sodium nitrate (Chile saltpetre) and potassium nitrates (Indian saltpetre).
- Nitrogen, the principle gas of atmosphere (78 % by volume) is separated industrially from liquid air by fractional distillation
- Ammonia is formed by the hydrolysis of urea.
- Nitric acid is prepared by heating equal amounts of potassium or sodium nitrate with concentrated sulphuric acid.
- In most of the reactions, nitric acid acts as an oxidising agent. Hence the oxidation state changes from +5 to a lower one. It doesn't yield hydrogen in its reaction with metals.
- The reactions of metals with nitric acid are explained in 3 steps as follows:
  - **Secondary reaction:** Nascent hydrogen produces the reduction products of nitric acid.
  - **Primary reaction:** Metal nitrate is formed with the release of nascent hydrogen
  - **Tertiary reaction:** The secondary products either decompose or react to give final products
- Phosphorus has several allotropic modification of which the three forms namely white, red and black phosphorus are most common.
- Yellow phosphorus is poisonous in nature and has a characteristic garlic smell. It glows in the dark due to oxidation which is called phosphorescence.
- Yellow phosphorus readily catches fire in air giving dense white fumes of phosphorus pentoxide.
- Phosphine is prepared by action of sodium hydroxide with white phosphorous in an inert atmosphere of carbon dioxide or hydrogen.
- Phosphine is used for producing smoke screen as it gives large smoke.

Notes:

- When a slow stream of chlorine is passed over white phosphorous, phosphorous trichloride is formed.
- Phosphorus trichloride: and Phosphorous pentachloride are used as a chlorinating agent
- Oxygen is paramagnetic. It exists in two allotropic forms namely dioxygen (O_2) and ozone or trioxygen (O_3).
- Ozone is commonly used for oxidation of organic compounds.
- Sulphur exists in crystalline as well as amorphous allotrophic forms. The crystalline form includes rhombic sulphur (α sulphur) and monoclinic sulphur (β sulphur). Amorphous allotropic form includes plastic sulphur (γ sulphur), milk of sulphur and colloidal sulphur.
- Sulphuric acid can be manufactured by lead chamber process, cascade process or contact process.
- When dissolved in water, it forms mono ($H_2SO_4.H_2O$) and dihydrates ($H_2SO_4.2H_2O$) and the reaction is exothermic.
- The halogens are present in combined form as they are highly reactive.
- Chlorine is manufactured by the electrolysis of brine in electrolytic process or by oxidation of HCl by air in Deacon's process.
- Chlorine is a strong oxidising and bleaching agent because of the nascent oxygen.
- When three parts of concentrated hydrochloric acid and one part of concentrated nitric acid are mixed, Aquaregia (Royal water) is obtained. This is used for dissolving gold, platinum etc...
- The hydrogen halides are extremely soluble in water due to the ionisation.
- Each halogen combines with other halogens to form a series of compounds called inter halogen compounds.
- Fluorine reacts readily with oxygen and forms difluorine oxide (F_2O) and difluorine dioxide (F_2O_2) where it has a -1 oxidation state.
- All the noble gases occur in the atmosphere.
- They are extremely stable and have a small tendency to gain or lose electrons.
- Sodium peroxide is very much known for its strong oxidizing property.
- The inertness of noble gases is an important feature of their practical uses.

UNIT 4

TRANSITION AND INNER TRANSITION ELEMENTS

- **Potassium dichromate ($K_2Cr_2O_7$)**

  > It is an orange red crystalline substance.

  > Chromium is in +6 oxidation state.
- **Chromyl Chloride test:**

  When potassium dichromate is heated with any chloride salt in the presence of conc H_2SO_4, orange red vapours of chromyl chloride (CrO_2Cl_2) is evolved.

  > This reaction is used for confirmation the presence of chloride ion in inorganic qualitative analysis.
- **Potassium Permanganate:** Exists in the form of dark purple crystal. Potassium manganate: Green in colour.
- Baeyer's Reagent: Cold dilute alkaline $KMnO_4$ is known as Baeyer's reagent.
- Actinoids: The fourteen elements following actinium i.e., from thorium (Th) to lawrentium (Lr) are called actinoids. Unlike the lanthanoids, all the actinoids are radioactive and most of them have short half lives.

  Examples: Thorium (Th), Uranium(U), Plutonium(Pu)
- IUPAC defines transition metal as an element whose atom has an incomplete *d* sub shell or which can give rise to cations with an incomplete *d* sub shell. They occupy the central position of the periodic table, between *s* and *p* block elements,
- *d*-Block elements composed of 3*d* series (4th period) Scandium to Zinc (10 elements), 4*d* series (5th period) Yttrium to Cadmium (10 elements) and 5*d* series (6th period) Lanthanum, Haffinium to mercury.
- The general electronic configuration of *d*- block elements can be written as [Noble gas] $(n{-}1)d^{1-10}ns^{1-2}$. Here, $n = 4$ to 7. In periods 6 and 7, the configuration includes $(n{-}2)f$ orbital; [Noble gas] $(n{-}2)f^{14}\ (n{-}1)d^{1\text{-}10}\ ns^{1-2}$.
- All the transition elements are metals. Similar to all metals the transition metals are good conductors of heat and electricity. Unlike the metals of Group-1 and group-2, all the transition metals except group 11 elements are hard.
- As we move from left to right along the transition metal series, melting point first increases as the number of unpaired *d* electrons available for metallic bonding increases, reach a maximum value and then decreases, as the *d* electrons pair up and become less available for

Notes:

bonding.

- Ionization energy of transition element is intermediate between those of *s* and *p* block elements. As we move from left to right in a transition metal series, the ionization enthalpy increases as expected.
- The first transition metal Scandium exhibits only +3 oxidation state, but all other transition elements exhibit variable oxidation states by loosing electrons from (*n*-1) *d* orbital and ns orbital as the energy difference between them is very small.
- In 3*d* series as we move from Ti to Zn, the standard reduction potential $\left(E^0_{M^{2+}/M}\right)$ value is approaching towards less negative value and copper has a positive reduction potential. i.e., elemental copper is more stable than Cu^{2+}.
- Most of the compounds of transition elements are paramagnetic. Magnetic properties are related to the electronic configuration of atoms.
- Many industrial processes use transition metals or their compounds as catalysts. Transition metal has energetically available *d* orbitals that can accept electrons from reactant molecule or metal can form bond with reactant molecule using its d electrons.
- Transition metals form a number of interstitial compounds such as TiC, $ZrH_{1.92}$, Mn_4N etc.
- Transition elements have a tendency to form coordination compounds with a species that has an ability to donate an electron pair to form a coordinate covalent bond.
- In the inner transition elements there are two series of elements.

  (1) Lanthanoids (previously called lanthanides)

  (2) Actinoids (previously called actinides)
- Lanthanoids have general electronic configuration [Xe] $4f^{2-14}\,5d^{0-1}\,6s^2$
- The common oxidation state of lanthanoides is +3
- As we move across 4*f* series, the atomic and ionic radii of lanthanoids show gradual decrease with increse in atomic number. This decrese in ionic size is called lanthanoid contraction.
- The electronic configuration of actinoids is not definite. The general valence shell electronic configuration of 5*f* elements is represented as [Rn] $5f^{2-14}\,6d^{0-2}\,7s^2$.
- Like lanthanoids, the most common state of actinoids is +3. In addition to that actinoids show variable oxidation states such as +2 , +3, +4 ,+5,+6 and +7

UNIT 5

COORDINATION CHEMISTRY

- **Types of complex:**

(i) Cationic complex

(ii) Anionic complex

(iii) Neutral complex

- Isomerism in coordination compounds:

(i)	Structural Isomerism	→ Ionisation
		→ Hydration
		→ Linkage
		→ Coordination

(ii)	Stereo Isomerism	→Geometrical Isomerism
		→ Optical Isomerism

- Crystal Field Theory (CFT) assumes that the bond between the ligand and the central metal atom is purely ionic. The bond is formed due to the electrostatic attraction between the electron rich ligand and the electron deficient metal.
- $\Delta E = h\nu = \dfrac{hc}{\lambda} = hc\bar{\nu}$
- The order of crystal field splitting power of various ligands.

  $I^- < Br^- < SCN^- < Cl^- < S^{2-} < F^- < OH^- \approx \text{urea} < ox^{2-} < H_2O$

  $< NCS^- < EDTA^{4-} < NH_3 < en < NO_2^- < CN^- < CO$
- CFSE $(\Delta E_o) = \{E_{LF}\} - \{E_{iso}\}$

  $= \{[n_{t_{2g}}(-0.4) + n_{e_g}(0.6)]\,\Delta_o + n_pP\} - \{n'_pP\}$
- Colour of the complex:

(i) Most of the transition metal complexes are coloured.

(ii) A substance exhibits colour when it absorbs the light of a particular wavelength in the visible region.

(iii) The complexes of central metal atom such as of Sc^{3+}, Ti^{4+}, Cu^+, Zn^{2+}, etc... are colourless. This is because the d-d transition is not possible in complexes with central metal having d^0 or d^{10} configuration.

- Metallic carbonyls:

(i) Classification based on the number of metal atoms present.

(a) Mononuclear carbonyls

Notes:

(b) Polynuclear carbonyls

(ii) Classification based on the structure:

(a) Non-bridged metal carbonyls

(b) Bridged carbonyls

- Stability constant (β):

$$\beta = \frac{[Cu\,(NH_3)_4]^{2+}}{[Cu^{2+}]\,[NH_3]^4}$$

- When two or more stable compounds in solution are mixed together and allowed to evaporate, in certain cases there is a possibility for the formation of double salts or coordination compounds. The double salts loose their identity and dissociates into their constituent simple ions in solutions, whereas the complex ion in coordination compound, does not loose its identity and never dissociate to give simple ions.
- According to werner, most of the elements exhibit, two types of valency namely primary valency and secondary valency and each element tend to satisfy both the valencies.In modern terminology, the primary valency is referred as the oxidation state of the metal atom and the secondary valency as the coordination number.
- Coordination entity is an ion or a neutral molecule, composed of a central atom, usually a metal and the array of other atoms or groups of atoms (ligands) that are attached to it.
- The central atom/ion is the one that occupies the central position in a coordination entity and binds other atoms or groups of atoms (ligands) to itself, through a coordinate covalent bond.
- The ligands are the atoms or groups of atoms bound to the central atom/ion. The atom in a ligand that is bound directly to the central metal atom is known as a donor atom.
- The complex ion of the coordination compound containing the central metal atom/ion and the ligands attached to it, is collectively called coordination sphere and are usually enclosed in square brackets with the net charge.
- The three dimensional spacial arrangement of ligand atoms/ions that are directly attached to the central atom is known as the coordination polyhedron (or polygon).
- The number of ligand donor atoms bonded to a central metal ion in a complex is called the coordination number of the metal.
- The oxidation state of a central atom in a coordination entity is defined as the charge it would bear if all the ligands were removed along with the electron pairs that were shared with the central atom.
- Linkage isomers arises when an ambidentate ligand is bonded to the central metal atom/ion through either of its two different donor atoms.
- Co-ordination isomers arises in the coordination compounds having both the cation and anion as complex ions. The interchange of one or more ligands between the cationic and the anionic coordination entities result in different isomers.
- Ionisation isomers arises when an ionisable counter ion (simple ion) itself can act as a ligand. The exchange of such counter ions with one or more ligands in the coordination entity will result in ionisation isomers.
- Geometrical isomerism exists in heteroleptic complexes due to different possible three dimensional spatial arrangements of the ligands around the central metal atom. This type of isomerism exists in square planer and octahedral complexes.
- Coordination compounds which possess chairality exhibit optical isomerism similar to organic compounds. The pair of two optically active isomers which are mirror images of each other are called enantiomers.
- Linus pauling proposed the Valance Bond Theory (VBT) which assumes that the bond formed between the central metal atom and the ligand is purely covalent. Bethe and Van vleck treated the interaction between the metal ion and the ligands as electrostatic and extended the Crystal Field Theory (CFT) to explain the properties of coordination compounds.

UNIT 6

SOLID STATE

- Bragg's equation:

$2d \sin\theta = n\,\lambda$

$$d = \frac{n\lambda}{2\sin\theta}$$

- Solids have definite volume and shape.
- Solids can be classified into the following two major types based on the arrangement of their constituents. (i) Crystalline solids (ii)Amorphous solids.
- A crystalline solid is one in which its constituents (atoms, ions or molecules), have an orderly arrangement extending over a long range.
- In contrast, in amorphous solids (In Greek, amorphous

Notes:

means no form) the constituents are randomly arranged.

- Crystalline solid is characterised by a definite orientation of atoms, ions or molecules, relative to one another in a three dimensional pattern. The regular arrangement of these species throughout the crystal is called a crystal lattice.
- A crystal may be considered to consist of large number of unit cells, each one in direct contact with its nearer neighbour and all similarly oriented in space.
- A unit cell is characterised by the three edge lengths or lattice constants *a*, *b* and *c* and the angle between the edges α, β and γ.
- There are seven primitive crystal systems; cubic, tetragonal, orthorhombic, hexagonal, monoclinic, triclinic and rhombohedral. They differ in the arrangement of their crystallographic axes and angles. Corresponding to the above seven, Bravis defined 14 possible crystal systems.
- In the simple cubic unit cell, each corner is occupied by an identical atoms or ions or molecules. And they touch along the edges of the cube, do not touch diagonally. The coordination number of each atom is 6.
- In a body centered cubic unit cell, each corner is occupied by an identical particle and in addition to that one atom occupies the body centre. Those atoms which occupy the corners do not touch each other, however they all touch the one that occupies the body centre. Hence, each atom is surrounded by eight nearest neighbours and coordination number is 8.
- In a face centered cubic unit cell, identical atoms lie at each corner as well as in the centre of each face. Those atoms in the corners touch those in the faces but not each other. The coordination number is 2.
- X-Ray diffraction analysis is the most powerful tool for the determination of crystal structure. The inter planar distance (d) between two successive planes of atoms can be calculated using the following equation form the X-Ray diffraction data $2d\sin\theta = n\lambda$.
- The structure of an ionic compound depends upon the stoichiometry and the size of the ions.generally in ionic crystals the bigger anions are present in the close packed arrangements and the cations occupy the voids. The ratio of radius of cation and anion $\left(\frac{r_{c^+}}{r_{A^+}}\right)$ plays an important role in determining the structure.
- Crystals always found to have some defects in the arrangement of their constituent particles.
- Schottky defect arises due to the missing of equal number of cations and anions from the crystal lattice.
- Frenkel defect arises due to the dislocation of ions from its crystal lattice. The ion which is missing from the lattice point occupies an interstitial position.
- Metal excess defect arises due to the presence of more number of metal ions as compared to anions.
- Metal deficiency defect arises due to the presence of less number of cations than the anions.

UNIT 7

CHEMICAL KINETICS

- Unit of rate = $\dfrac{\text{Unit of concentration}}{\text{Unit of time}}$
- $k = \dfrac{2.303}{t}\log\left[\dfrac{[A_0]}{[A]}\right]$
- Half life period of a reaction:

  $t_{1/2} = \dfrac{0.6932}{k}$
- Arrhenius equation:

  Arrhenius suggested that the rates of most reactions vary with temperature in such a way that the rate constant is directly proportional to

  $e^{-\left(\frac{E_a}{RT}\right)}$

  and he proposed a relation between the rate constant and temperature.

  $k = Ae^{-\left(\frac{E_a}{RT}\right)}$

  A = the frequency factor,

  R = the gas constant,

  E_a = the activation energy
- Chemical kinetics is the study of the rate and the mechanism of chemical reactions, proceeding under given conditions of temperature, pressure, concentration etc.
- The change in the concentration of the species involved in a chemical reaction per unit time gives the rate of a reaction.

Notes:

- The rate of the reaction, at a particular instant during the reaction is called the instantaneous rate. The shorter the time period, we choose, the closer we approach to the instantaneous rate,
- The rate represents the speed at which the reactants are converted into products at any instant.
- The rate constant is a proportionality constant and It is equal to the rate of reaction, when the concentration of each of the reactants in unity
- Molecularity of a reaction is the total number of reactant species that are involved in an elementary step.
- The half life of a reaction is defined as the time required for the reactant concentration to reach one half its initial value. For a first order reaction, the half life is a constant i.e., it does not depend on the initial concentration.
- According to collision theory, chemical reactions occur as a result of collisions between the reacting molecules.
- Generally, the rate of a reaction increase with increasing temperature. However, there are very few exceptions. The magnitude of this increase in rate is different for different reactions. As a rough rule, for many reactions near room temperature, reaction rate tends to double when the temperature is increased by 10°C.
- According to Arrhenius, activation energy of the reaction is the minimum energy that a molecule must have to possess to react.
- The rate of a reaction is affected by the following factors.
  1. Nature and state of the reactant
  2. Concentration of the reactant
  3. Surface area of the reactant
  4. Temperature of the reaction
  5. Presence of a catalyst

UNIT 8

IONIC EQUILIBRIUM

- Acid tastes sour, turns the blue litmus to red and reacts with metals such as zinc and produces hydrogen gas.
- **Lewis acids:** Electron deficient molecules such as BF_3, $AlCl_3$, BeF_2 etc.
- **Lewis bases:** Molecules with one (or) more lone pairs of electrons.

  NH_3, H_2O, R–O–H, R–O–R, R-NH_2
- Relation between p^H and p^{OH}:

  $p^H = -\log_{10} [H_3O^+]$

  $p^{OH} = -\log_{10} [OH^-]$

  $p^H + p^{OH} = pk_W$
- Ostwald's dilution law:

$$\alpha = \sqrt{\frac{k_a}{C}}$$

- Common ion effect:

  The dissociation of a weak acid (CH_3COOH) is suppressed in the presence of a salt (CH_3COONa) containing an ion common to the weak electrolyte. It is called the common ion effect.
- Buffer solution: Which consists of a mixture of a weak acid and its conjugate base (or) a weak base and its conjugate acid.
- Types of buffer solution:

  There are two types of buffer solutions.

(i) Acidic buffer solution: a solution containing a weak acid and its salt.

Example: solution containing acetic acid and sodium acetate

(ii) Basic buffer solution: a solution containing a weak base and its salt.

Example: Solution containing NH_4OH and NH_4Cl

- **Buffer capacity** and buffer index is defined as the number of gram equivalents of acid or base added to 1 litre of the buffer solution to change its pH by unity.

$$\beta = \frac{dB}{d(pH)}$$

- **Henderson – Hasselbalch equation**

$$\Rightarrow p^H = pK_a + \log\frac{[salt]}{[acid]}$$

$$\Rightarrow p^{OH} = pK_b + \log\frac{[salt]}{[base]}$$

- **Neutralization:** When an acid reacts with a salt and water are formed and the reaction is called neutralization.

$$K_h = \sqrt{\frac{K_w}{K_a \times C}}$$

- **Solubility product:**

(i) Ionic product > Ksp , precipitation will occur and the solution is super saturated.

(ii) Ionic product < Ksp , no precipitation and the solution

Notes:

is unsaturated.

(iii) Ionic product = Ksp , equilibrium exist and the solution is saturated.

- Hydrolysis of Salt of strong base and weak acid
  $K_h.K_a = K_w$
  $$pH = 7 + \frac{1}{2} pK_a + \frac{1}{2} \log C$$
- Hydrolysis of salt of strong acid and weak base
  $K_h.K_a = K_w$
  $$pH = 7 - \frac{1}{2} pK_b - \frac{1}{2} \log C.$$
- Hydrolysis of Salt of weak acid and weak base
  $K_a.K_b.K_h = K_w$
  $$pH = 7 + \frac{1}{2} pK_a - \frac{1}{2} pK_b.$$
- The solubility product of a compound is defined as the product of the molar concentration of the constituent ions, each raised to the power of its stoichiometric co-efficient in a balanced equilibrium equation.
- According to Arrhenius, an acid is a substance that dissociates to give hydrogen ions in water.
- According to Lowry and Bronsted concept, an acid is defined as a substance that has a tendency to donate a proton to another substance and base is a substance that has a tendency to accept a proton form other substance.
- According to Gilbert . N. Lewis , an acid is a species that accepts an electron pair while base is a species that donates an electron pair.
- Ionic product (ionic product constant) of water $(K_w)=[H_3O^+]\ [OH^-]$
- pH of a solution is defined as the negative logarithm of base 10 of the molar concentration of the hydronium ions present in the solution.
  $pH = -\log_{10} [H_3O^+]$
- When dilution increases, the degree of dissociation of weak electrolyte also increases. This statement is known as **Ostwald's dilution Law.**

UNIT 9

ELECTRO CHEMISTRY

- A conductivity cell consists of two electrodes immersed in an electrolytic solution.
  $$I \propto V \text{ (or) } I = \frac{V}{R} \Rightarrow V = IR$$
- **Resistivity** (ρ): The resistance of such an electrolytic solution is also directly proportional to the length (l) and inversely proportional to the cross sectional area (A).
  $$P \propto \frac{l}{A}$$
  $$R = \rho \frac{l}{A}$$
- **Cell constant:** The ratio $\left(\frac{l}{A}\right)$
- **Conductivity:** It is more convenient to use conductance rather than resistance. The reciprocal of the resistance $\left(\frac{1}{R}\right)$ gives the conductance of an electrolytic solution.
  The SI unit of conductance is Siemen (S) .
  $$C = \frac{1}{R}$$
- **Molar Conductivity:** Let us imagine a conductivity cell in which the electrodes are separated by 1m and having V m^3 of electrolytic solution which contains 1 mole of electrolyte. The conductance of such a system is called the molar conductance (Λ_m)
- **Specific conductance (or) Conductivity of an electrolyte:**
  $$\kappa = \frac{1}{R}\left(\frac{l}{A}\right)$$
- **Debye - Huckel and Onsager equation:**
  The influence of ion-ion interactions on the conductivity of strong electrolytes was studied by Debye and Huckel.
  $$\Delta_m = \Delta_{m0} - (A + B\Delta_{m0})\sqrt{C}$$
- **Kohlraush's law:** At infinite dilution, the limiting molar conductivity of an electrolyte is equal to the sum of the limiting molar conductivities of its constituent ions.
  $$\left(\lambda_m^0\right)_{NaCl} = \left(\lambda_m^0\right)_{Na^+} + \left(\lambda_m^0\right)_{Cl^-}$$
- **Electrochemical cell:**

(i) **Galvanic Cell (Voltaic cell):** It is a device in which a spontaneous chemical reaction generates an electric current i.e., it converts chemical energy into electrical energy. It is commonly known as a battery.

(ii) **Electrolytic cell:** It is a device in which an electric

Notes:

current from an external source drives a nonspontaneous reaction i.e., it converts electrical energy into chemical energy.

- **Degree of dissociation of weak electrolytes:**

$$K_a = \frac{\lambda_m^2 C}{\lambda_m^0 (\lambda_m^0 - \lambda_m)}$$

- **emf of a cell:** The force that pushes the electrons away from the anode and pulls them towards cathode is called the electromotive force (emf).
- The cell voltage depends on the nature of the electrodes.

Charge of one elctron = 1.602×10^{-19} C

Charge of one mole of elctron

$$= 6.023 \times 10^{23} \times 1.602 \times 10^{-19} \text{ C}$$
$$= 96488 \text{ C}$$
$$\text{i.e., } 1 \text{ F} = 96500 \text{ C}$$

- $\Delta G = \Delta G^0 + RT \ln Q_C$
- **Nernst equation**:

$$E_{\text{cell}} = E^0_{\text{cell}} - \frac{2.303 \text{ RT}}{\text{nF}} \log \frac{(C)^l (D)^m}{(A)^x (B)^y}$$

- **Electrolysis**: Electrolysis is a process in which the electrical energy is used to cause a non-spontaneous chemical reaction to occur. The energy is often used to decompose a compound into its elements.
- **Faraday's I law:** The mass of the substance (m) liberated at an electrode during electrolysis is directly proportional to the quantity of charge (Q) passed through the cell.

$m \propto Q$

$m \propto It$

$m = ZIt$ (Z-electro chemical equivalent)

- **Batteries:** Batteries are indispensable in the modern electronic world.
- **Recharge of the cell:** E.g. Lithium-ion Battery. During recharge process, the role of anode and cathode is reversed and H_2SO_4 is regenerated.
- **Fuel cell:** The galvanic cell in which the energy of combustion of fuels is directly converted into electrical energy is called the fuel cell.
- **Corrosion:** The metal is oxidised by oxygen in presence of moisture. This redox process which causes the deterioration of metal is called corrosion.
- As the corrosion of iron causes damages to our buildings, bridges etc... it is important to know the chemistry of rusting and how to prevent it. Rusting of iron is an electrochemical process.

UNIT 10
SURFACE CHEMISTRY

- **Adsorption:** Adsorbent is the material on which adsorption takes place. Adsorbed substance is called an adsorbate.
- **Adsorption** is a spontaneous process and it is always accompanied by decrease in free energy. When DG reaches zero, the equilibrium is attained.

$\Delta G = \Delta H - T\Delta S$ where ΔG is Change in Free energy.

ΔH = Change in enthalpy and ΔS = Change in entropy.

- **Factors affecting adsorption:**

(a) Nature of adsorbent

(b) Nature of adsorbate

(c) Pressure

(d) Concentration at a given temperature.

- **Chromatography:** The chromatographic technique is applied for separation of components in a mixture.
- **Catalyst:** catalyst is defined as a substance which alters the rate of chemical reaction without itself undergoing chemical change. The phenomenon which involves the action of a catalyst is called catalysis.
- **Homogeneous catalysis:** The reactants, products and catalyst are present in the same phase.
- **Heterogeneous catalysis:** In a reaction, the catalyst is present in a different phase. It is not present in the same phase as that of reactants or products.
- **Promoters:** In a catalysed reaction the presence of a certain substance increases the activity of a catalyst. Such a substance is called a promoter.
- **Active centres:** The surface of a catalyst is not smooth. It bears steps, cracks and corners. Hence the atoms on such locations of the surface are co-ordinatively unsaturated. So, they have much residual force of attraction. Such sites are called active centres. So, the surface carries high surface free energy.
- **Pepsin:** The peptide glycyl L-glutamyl L-tyrosin is hydrolysed by an enzyme called pepsin.
- **Lyophillic colloids:** Definite attractive force or affinity exists between dispersion medium and dispersed phase. Examples: sols of protein and starch.
- **Lyophobic colloids:** No attractive force exists between

Notes:

the dispersed phase and dispersion medium. They are less stable and precipitated readily, but can not be produced again by just adding the dispersion medium. They themselves undergo coagulation after a span of characteristic life time. They are called irreversible sols

Examples: sols of gold, silver, platinum and copper.

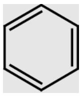
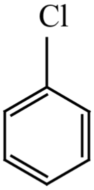
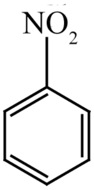
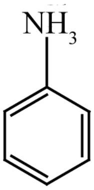
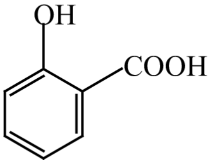
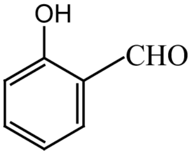
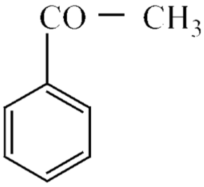
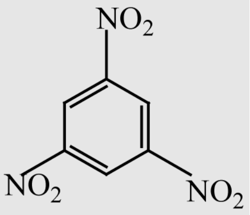
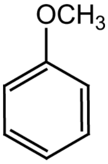
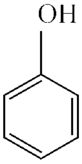
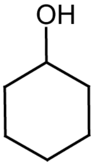
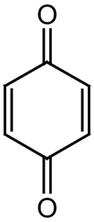
- **Alkali hydroxide** is added as an stabilising agent for the colloidal solution.
- **Peptisation:** By addition of suitable electrolytes, precipitated particles can be brought into colloidal state.
- **Ultrafiltration:** The separation of sol particles from electrolyte by filteration through an ultrafilter is called ultrafiltration. Collodion is 4% solution of nitrocellulose in a mixture of alcohol and water.
- **Tyndall effect:** When light passes through colloidal solution, it is scattered in all directions. This is called as Tyndall effect.
- **Brownian movement:** The colloidal sol particles are continuously bombarded with the molecules of the dispersion medium and hence they follow a zigzag, random, continuous movement.
- **Helmholtz double layer:** At the boundary separating the two electrical double layers are setup.
- **Emulsions:** The process of preparation of emulsion by the dispersal of one liquid in an another liquid.
- **Inversion of Phase:** The change of W/O emulsion into O/W emulsion is called inversion of phases.

UNIT 11, 12, 13

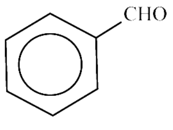
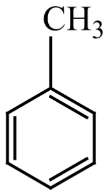
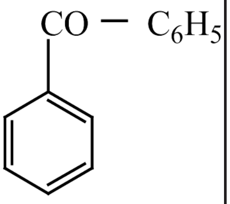
HYDROXY COMPOUNDS AND ETHERS

CARBONYL COMPOUNDS AND CARBOXYLIC ACIDS

ORGANIC NITROGEN COMPOUNDS

1.	 Benzene	2.	 ChloroBenzene
3.	 NitroBenzene	4.	 Aniline
5.	 Salicylic acid	6.	 Salicylaldehyde
7.	 Acetophenone	8.	CH_3COOH Acetic acid
9.	CH_3CH_2OH Ethanol	10.	 1,3,5-Trinitrobenzene
11.	 Anisole	12.	 Phenol
13.	 Cyclohexanol	14.	 1,4 Benzoquinone

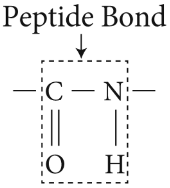
Notes:

15.	 Benzaldehyde	16.	 Methylbenzene
17.	 Benzophenone	18	CH_3CHO Acetaldehyde
19.	$CH_3–CO–CH_3$ Acetone	20.	$CH_3–CONH_2$ Acetamide

UNIT 14

BIOMOLECULES

- **Carbohydrates:** Carbohydrates are the most abundant organic compounds in every living organism. They are also known as saccharides.
- **Glucose:** $C_6H_{12}O_6$; simple sugar. It is also known as blood sugar.
- **Fructose:** Same molecular formula as glucose. It is levorotatory and a ketohexose. It is also called as fruit sugar.
- $\text{Sucrose} \xrightarrow{\text{Invertase}} \text{Glucose + Fructose}$
- **Lactose** → Milk sugar
- **STARCH:** Starch is used for energy storage in plants. Starch contains about 20 % of amylose and about 80% of amylocpectin.
- **Cellulose:** Cellulose is the major constituent of plant cell walls. Cotton is almost pure cellulose. Cellulose is used extensively in the manufacturing paper, cellulose fibres, rayon explosive, (Gun cotton – Nitrated ester of cellulose) and so on.
- Human cannot use cellulose as food because our digestive systems do not contain the necessary enzymes.
- **Proteins:** polymers of α-amino acids.
- **Amino acids:** Amino acids are compounds which contain an amino group and a carboxylic acid group.
- **Zwitter ions:** In aqueous solution the proton from carboxyl group can be transferred to the amino group of an amino acid leaving these groups with opposite charges. Despite having both positive and negative charges this molecule is neutral and has amphoteric behaviour. These ions are called zwitter ions.
- **Peptide bond:** The amino acids are linked covalently by peptide bonds. The carboxyl group of the first amino acid react with the amino group of the second amino acid to give an amide linkage between these amino acids. This amide linkage is called peptide bond.



- **Enzymes:** There are many biochemical reactions that occur in our living cells. All these reactions are catalysed by special proteins called enzymes. They are biocatalysts that catalyse a specific biochemical reaction.
- **Lipids:** Lipids are organic molecules that are soluble in organic solvents such as chloroform and methanol and insoluble in water.
- **Vitamins:** They are small organic compounds that cannot be synthesised by our body but are essential for certain functions.
- **Fat soluble vitamin:** A, D, E & K
- **Water soluble vitamins:** B (B_1, B_2, B_3, B_5, B_6, B_7, B_9 and B_{12}) and C.
- **Nucleic acids:** Chromosomes and are made up of proteins and another type of biomolecules called nucleic acids.
- **Types of RNA:**
  (i) Ribosomal RNA (rRNA)
  (ii) Messenger RNA (mRNA)
  (iii) Transfer RNA (tRNA)
- **DNA finger printing:** DNA fingerprinting is (also called DNA typing or DNA profiling) unique for every person and can be extracted from traces of samples from blood, saliva, hair etc… By using this method we can detect the individual specific variation in human DNA.
- **HORMONES:** Hormone is an organic substance (e.g. a peptide or a steroid) that is secreted by one tissue. It limits the blood stream and induces a physiological response (e.g. growth and metabolism) in other tissues.

Notes:

UNIT 15
CHEMISTRY IN EVERYDAY LIFE

- **Drug:** Drug is a substance that is used to modify or explore physiological systems or pathological states for the benefit of the recipient.
- **Medicine:** The drug which interacts with macromolecular targets such as proteins to produce a therapeutic and useful biological response is called medicine.
- **Chemotherapy:** The specific treatment of a disease using medicine is known as chemotherapy.
- **Antibiotic drugs:** amoxicillin, ampicillin, cefixime, cefpodoxime, erythromycin, tetracycline etc..
- **Antihypertensive drugs:** propranolol, atenolol, metoprolol succinate, amlodipine etc…
- **PABA:** p-aminobenzoic acid.
- **DHPS:** dihydropteroate synthase.
- **Antagonists:** If we want to block a message, a drug that binds to the receptor site should inhibit its natural function. Such drugs are called antagonists.
- **Agonists:** In contrast, there are drugs which mimic the natural messenger by switching on the receptor. These type of drugs are called agonists and are used when there is lack of chemical messenger.
- **Tranquilizers:** They are neurologically active drugs. Acts on the central nervous system by blocking the neurotransmitter dopamine in the brain
- **Anaesthetics:**

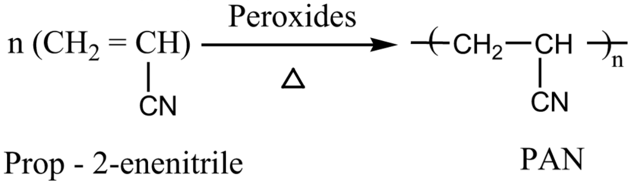
(i) **Local anaesthetics:** It causes loss of sensation, in the area in which it is applied without losing consciousness. They block pain perception that is transmitted via peripheral nerve fibres to the brain.

(ii) General anaesthetics: Cause a controlled and reversible loss of consciousness by affecting central nervous system.

- **Antacids:** Neutralize the acid in the stomach that causes acidity. **Uses:** To relieve symptoms such as burning sensation in the chest/ throat area (heart burns) caused by acid reflux.
- **Antiseptics:** To reduce the risk of infection during surgery and other procedures.
- **Disinfectants:** Stop or slow down the growth of microorganisms – Generally used on inanimate objects.
- **Antioxidants:** Antioxidants are substances which retard the oxidative deteriorations of food. Food containing fats and oils is easily oxidised and turn rancid.
- **Artificial sweetening agents:** Synthetic compounds which imprint a sweet sensation and possess no or negligible nutritional value are called artificial sweeteners. Eg. Saccharin, Aspartame, sucralose, alitame etc…
- **Soaps:** Soaps are made from animal fats or vegetable oils.
- **Detergents:** They are superior to soaps as they can be used even in hard water and in acidic conditions. The cleansing action of detergents are similar to the cleansing action of soaps.
- **PVC:** Poly vinyl chloride.
- **Polymerisation:** The process of forming a very large, high molecular mass polymer from small structural units i.e., monomer is called polymerisation.
- **Nylon – 6,6:** Nylon – 6,6 can be prepared by mixing equimolar adipic acid and hexamethylene – diamine to form a nylon salt which on heating eliminate a water molecule to form amide bonds.
- **Vulcanization:** Cross linking of Rubber.
- **Synthetic rubber:** Polymerisation of certain organic compounds such as buta-1,3-diene or its derivatives gives rubber like polymer with desirable properties like stretching to a greater extent etc., such polymers are called synthetic rubbers.
- **Polythene:**

$$n\,\underset{\text{ethene}}{CH_2 = CH_2} \xrightarrow[1000\ \text{atm}]{200^{\circ} - 300^{\circ}C} \underset{\text{Polythene}}{-\!\!\left(CH_2-CH_2\right)_n\!\!-}$$

- **PAN:** Polyacrylonitrile



$$\underset{\text{Prop - 2-enenitrile}}{n\,(CH_2 = \underset{\underset{CN}{|}}{CH})} \xrightarrow[\Delta]{\text{Peroxides}} \underset{\text{PAN}}{-\!\!\left(CH_2-\underset{\underset{CN}{|}}{CH}\right)_n\!\!-}$$

- **Dacron:** It is used in blending with cotton or wool fibres and as glass reinforcing materials in safety helmets.
- **Buna-N:** It is a co-polymer of acrylonitrile and buta-1,3-diene. It is used in the manufacture of hoses and tanklinings.
- **Buna-S:** It is a co-polymer. It is obtained by the polymerisation of buta-1,3-diene and styrene in the ratio 3:1 in the presence of sodium.

****

Notes:

PROBLEM - QUESTIONS & ANSWERS

UNIT - 6

SOLID STATES

FORMULA:

- Number of atoms in a sc unit cell $= \left(\frac{NC}{8}\right) = \left(\frac{8}{8}\right) = 1$
- Number of atoms in a bcc unit cell

  $$= \left(\frac{N_C}{8}\right) + \left(\frac{N_b}{1}\right) = \left(\frac{8}{8} + \frac{1}{1}\right) = (1+1) = 2$$

- Number of atoms in a fcc unit cell

  $$= \left(\frac{N_C}{8}\right) + \left(\frac{N_f}{2}\right) = \left(\frac{8}{8} + \frac{6}{2}\right) = (1+3) = 4$$

- Edge length of the unit cell $= d = \frac{n\lambda}{2\sin\theta}$
- Bragg's equation: $2d\sin\theta = n\lambda$
- Density of the unit cell $\left[\rho = \frac{nM}{a^3 N_A}\right]$
- Mass of one atom $m = \frac{M}{N_A}$
- Mass of one atom $= \frac{\text{molar mass (g mol}^{-1})}{\text{Avagadro number } 6.023\times10^{23}(\text{mol}^{-1})}$

$$\left\{\begin{array}{l}\text{Packing efficiency for}\\ \text{simple cubic arrangement}\end{array}\right\} = \frac{\left\{\begin{array}{l}\text{Total volume occupied by}\\ \text{spheres in a unit cell}\end{array}\right\}}{\text{Volume of the unit cell}} \times 100$$

- Packing fraction of sc arrangement $= \frac{\left(\frac{\pi a^3}{6}\right)}{(a^3)} \times 100$
- Packing fraction of bcc arrangement $= \frac{\left(\frac{\sqrt{3}\,\pi\,a^3}{8}\right)}{(a^3)} \times 100$
- Packing fraction of fcc arrangement $= \frac{\left(\frac{\sqrt{2}\,\pi\,a^3}{6}\right)}{(a^3)} \times 100$

Notes:

UNIT - 11, 12, 13

Naming reactions:

1. Hydroxy derivatives and ethers.

(i) Anti-Markownikoff's product

Step (i)

$6\,CH_3–CH=CH_2 + B_2H_6 \longrightarrow 2(CH_3–CH_2–CH_2)_3\,B$

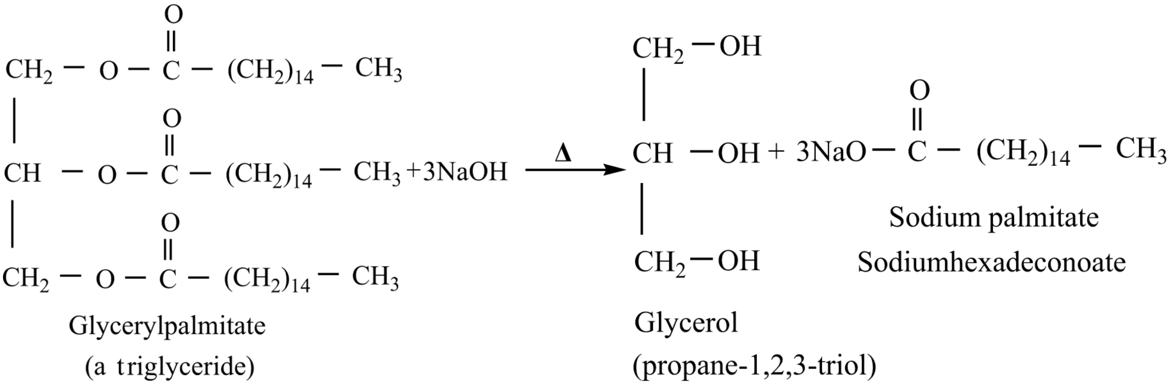
Propene Diborane Tripropylborane

Step (ii)

$(CH_3–CH_2–CH_2)_3B + 3\,H_2O_2 \longrightarrow 3CH_3–CH_2–CH_2–OH + B(OH)_3$

Tripropylborane Hydrogen peroxide Propan-1-ol

(2) Saponification Reaction:



$$CH_2(OCO(CH_2)_{14}CH_3)–CH(OCO(CH_2)_{14}CH_3)–CH_2(OCO(CH_2)_{14}CH_3) + 3NaOH \xrightarrow{\Delta} CH_2(OH)–CH(OH)–CH_2(OH) + 3NaO–\overset{O}{\overset{\|}{C}}–(CH_2)_{14}–CH_3$$

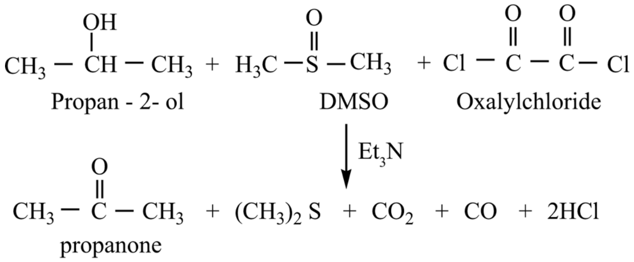
Glycerylpalmitate (a triglyceride)

Glycerol (propane-1,2,3-triol)

Sodium palmitate Sodiumhexadeconoate

(3) Swern oxidation:

In this method Dimethyl sulfoxide (DMSO) is used as the oxidising agent.



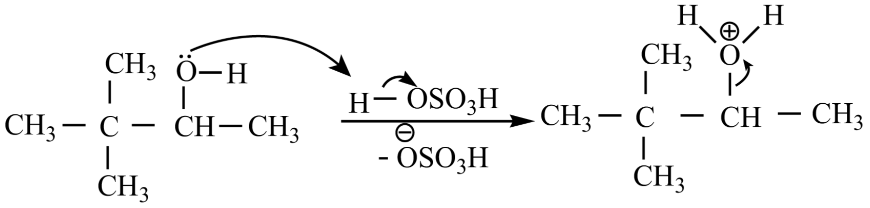
$$CH_3-\overset{OH}{\overset{|}{C}H}-CH_3 + H_3C-\overset{O}{\overset{\|}{S}}-CH_3 + Cl-\overset{O}{\overset{\|}{C}}-\overset{O}{\overset{\|}{C}}-Cl \xrightarrow{Et_3N} CH_3-\overset{O}{\overset{\|}{C}}-CH_3 + (CH_3)_2S + CO_2 + CO + 2HCl$$

Propan - 2- ol DMSO Oxalylchloride

propanone

(4) Saytzeff's rule:

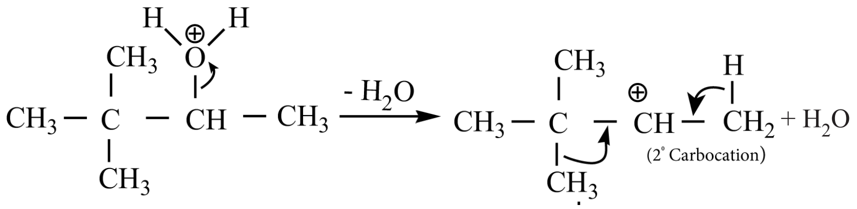
Step I:



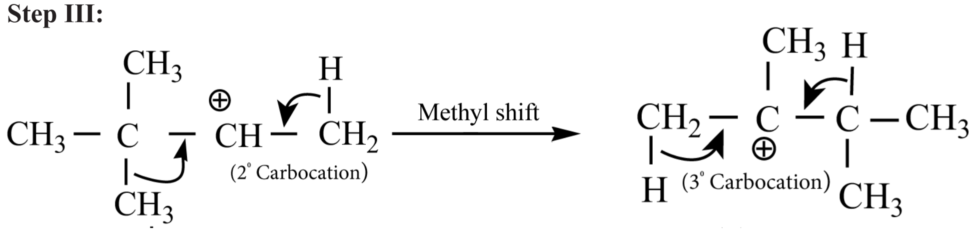
3,3 – dimethyl – 2- butanol

Notes:

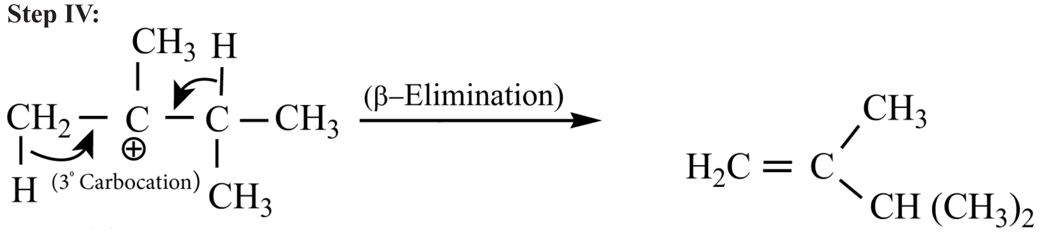
Step II:



Step III:

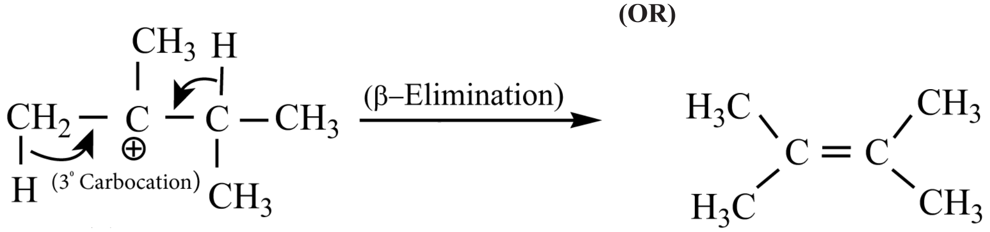


Step IV:



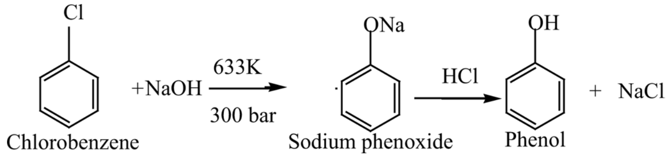
2,3 - dimethylbut -1-ene

(OR)



2,3 - dimethylbut - 2- ene

(5) Dows process:



(6) Schotten Baumann

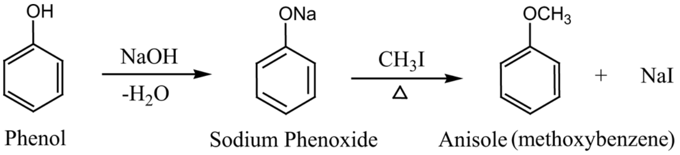
$$\underset{\text{Phenol}}{C_6H_5OH} + \underset{\text{Acetyl chloride}}{CH_3COCl} \xrightarrow[Py]{NaOH} \underset{\text{Phenyl ethanoate}}{CH_3\text{-}COOC_6H_5} + HCl$$

(or)

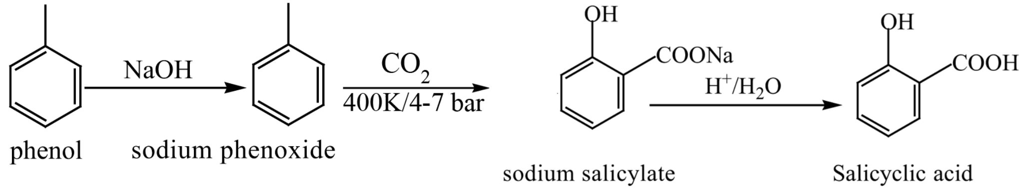
$$\underset{\text{Phenol}}{C_6H_5OH} + \underset{\text{Benzoyl chloride}}{C_6H_5COCl} \xrightarrow[Py]{NaOH} \underset{\text{Phenyl benzoate}}{C_6H_5\text{- }COOC_6H_5} + HCl$$

Notes:

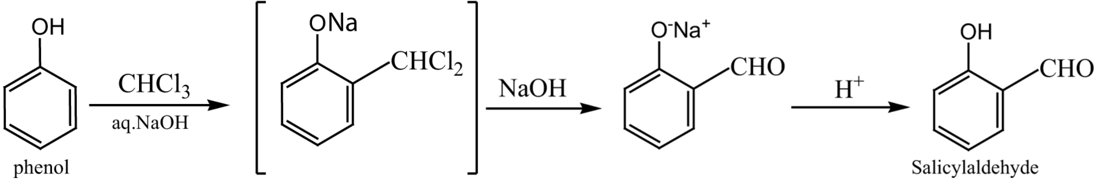
(7) Williamson ether synthesis:



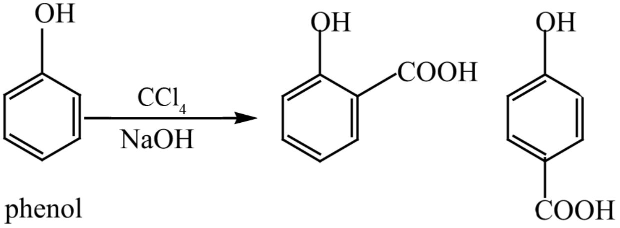
(8) Kolbe's (or) Kolbe Schmit reaction:



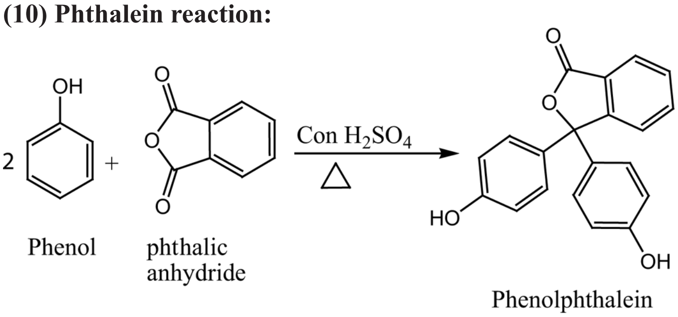
(9) Riemer-Tiemann Reaction:



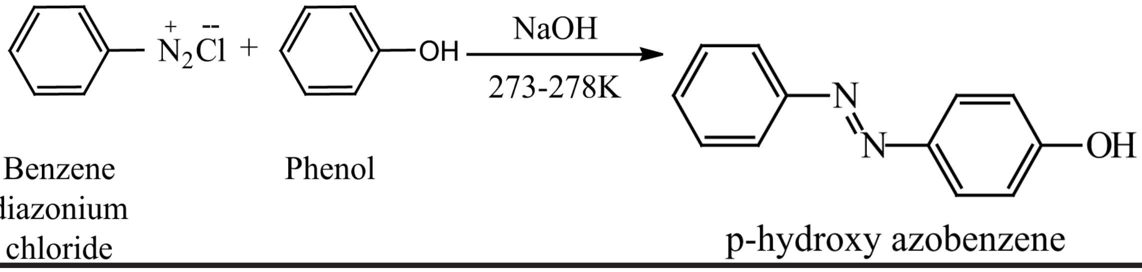
With CCl_4 and NaOH hydroxy benzoic acid is formed.



(10) Phthalein reaction:

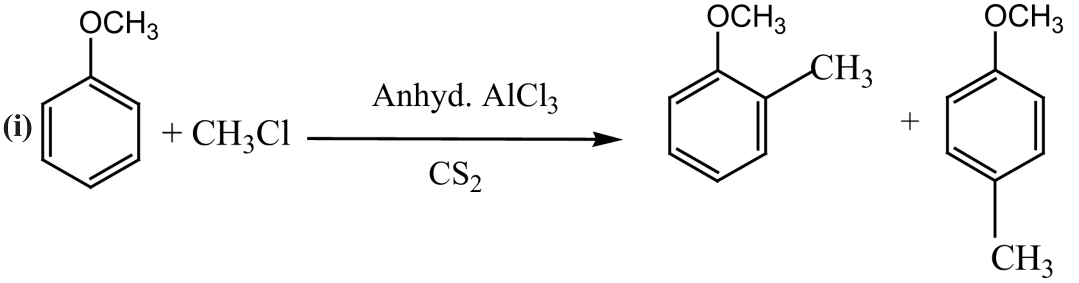


(11) Coupling reaction:



Notes:

(12) Friedel Craft's reaction:

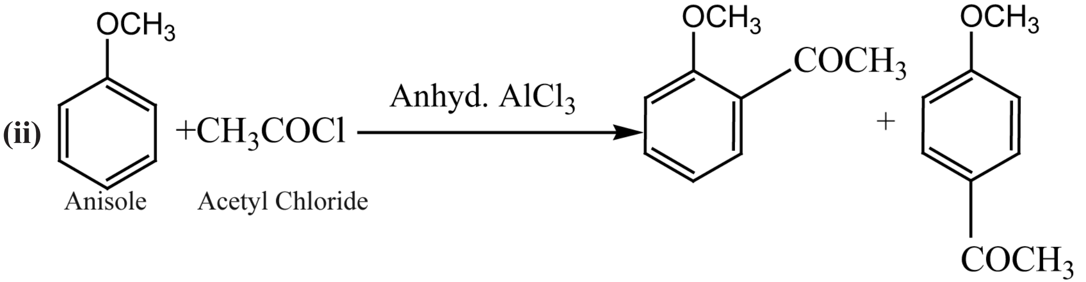


2-methoxy toluene(minor)

4-methoxy toluene (major)

O-methyl anisole

p-methyl anisole



2-methoxy acetophenone (minor)

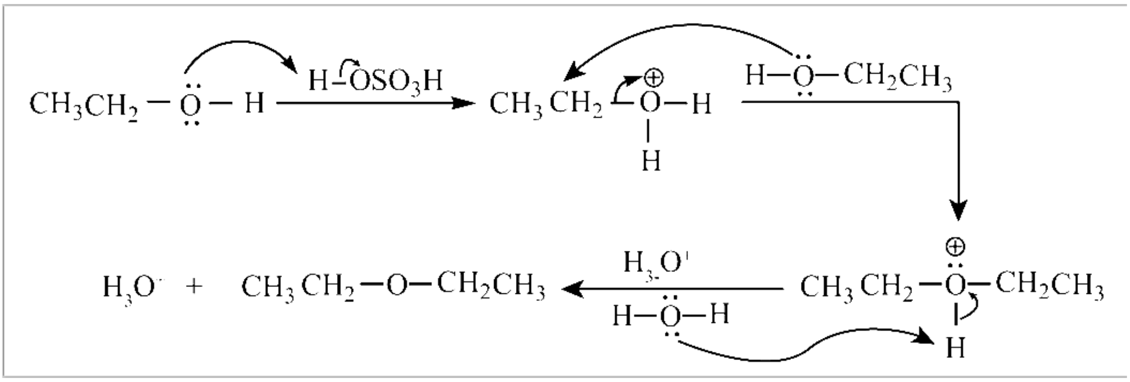
4-methoxy Acetophenone (major)

(13) Inter molecular dehydration of alcohol

Mechanism

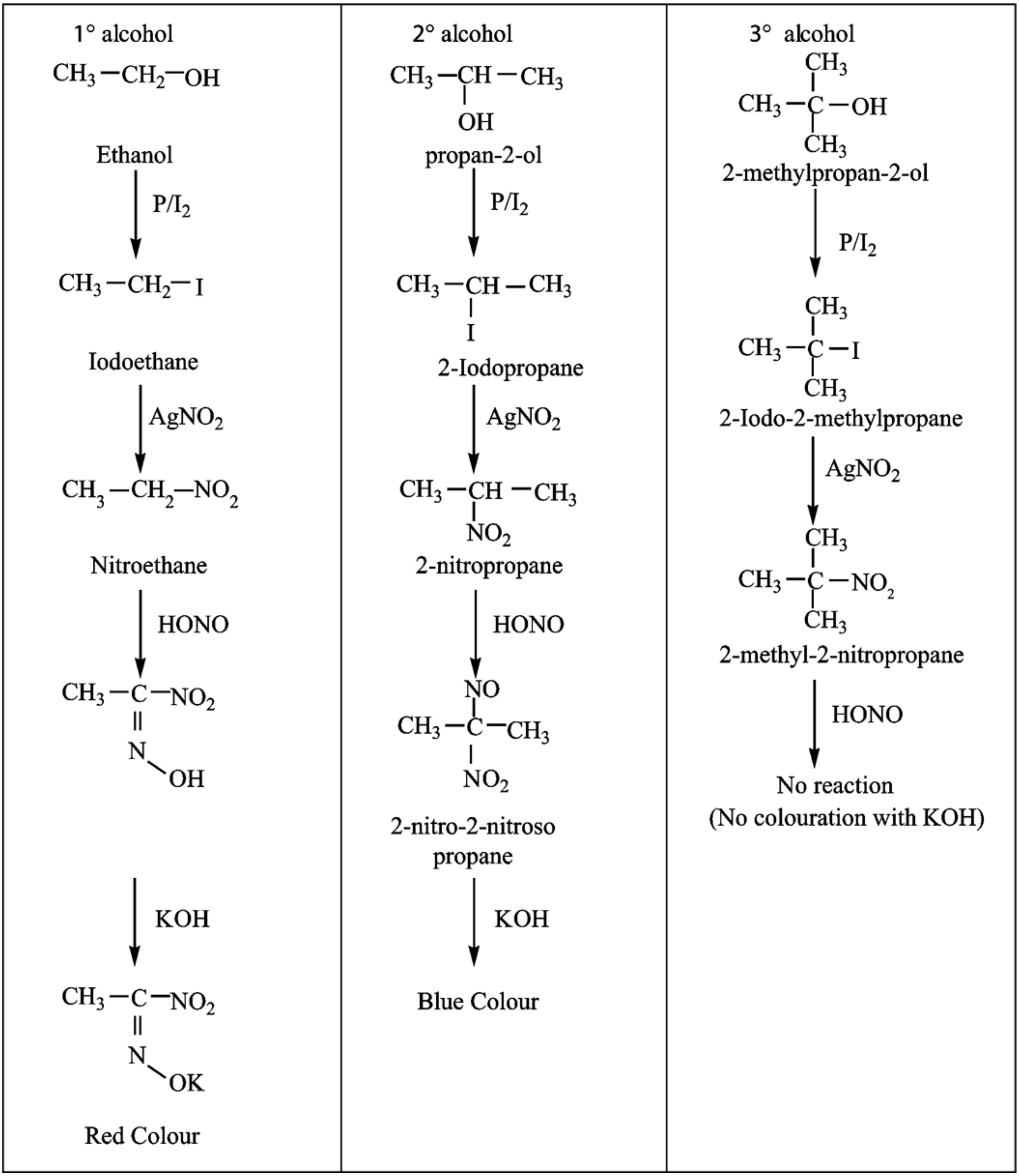
$$2CH_3\text{-}CH_2\text{-}OH \xrightarrow[413K]{H_2SO_4} CH_3\text{-}CH_2\text{-}O\text{-}CH_2\text{-}CH_3$$

ethanol diethylether



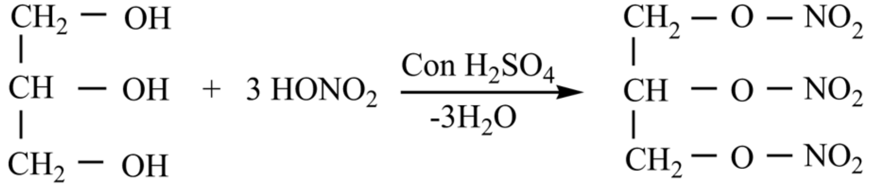
Notes:

(14) Victor Mayer's test:



Notes:

(15) TNG Preparation [Reaction of Glycerol]



$$\begin{array}{l} CH_2-OH \\ | \\ CH-OH \\ | \\ CH_2-OH \end{array} + 3\,HONO_2 \xrightarrow[-3H_2O]{Con\ H_2SO_4} \begin{array}{l} CH_2-O-NO_2 \\ | \\ CH-O-NO_2 \\ | \\ CH_2-O-NO_2 \end{array}$$

Propan - 1,2,3 - triol
glycerol

1,2,3 - trinitroxy propane

(16) LucasTest: [For 1°, 2°, 3°, alcohol]

Lucas test is to distinguish between primary, secondary and tertiary alcohols. Lucas reagent is a mixture of **anhydrous $ZnCl_2$ and conc. HCl.**

For 3° alcohol: Tertiary alcohols reacts immediately to form turbidity which is insoluble in the medium.

$$(CH_3)_3C{-}OH \xrightarrow[HCl]{ZnCl_2} (CH_3)_3C{-}Cl + H_2O$$

2-chloro-2methyl propane

For 2° alcohol: Secondary alcohols gives turbidity with in 5 to 10 minutes.

$$(CH_3)_2CHOH \xrightarrow[HCl]{ZnCl_2} (CH_3)_2CHCl + H_2O$$

Propan-2-ol 2-Chloro propane

For 1° alcohol:

Primary alcohol has no reaction at room temperature.

$$CH_3CH_2OH \xrightarrow[HCl]{ZnCl_2} \text{No reaction at room temperature}$$

(17) Nucleophilic substitution reactions of alcohols:

S_N2 Mechanism

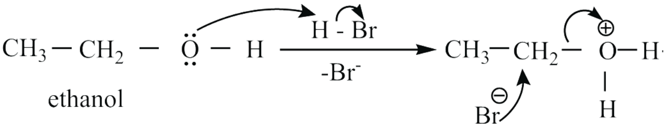
E.g.:

$$CH_3-CH_2-OH \xrightarrow[-\ H_2O]{HBr} CH_3-CH_2-Br$$

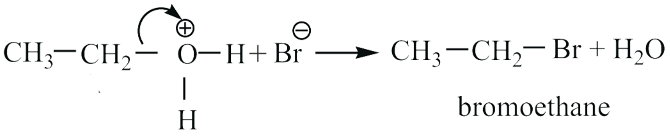
Ethanol Bromoethane

Mechanism:

Step-I



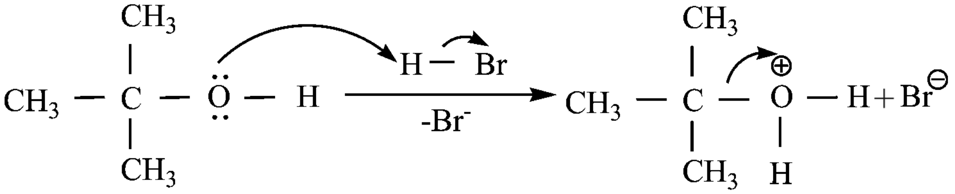
Step-II



Notes:

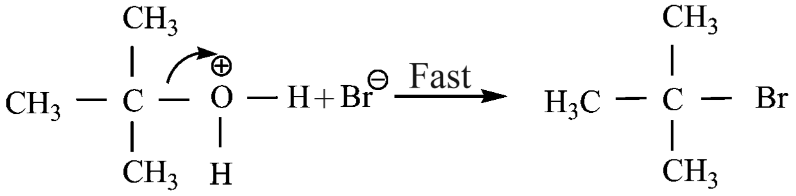
(18) S_N1 Mechanism

Step-I:



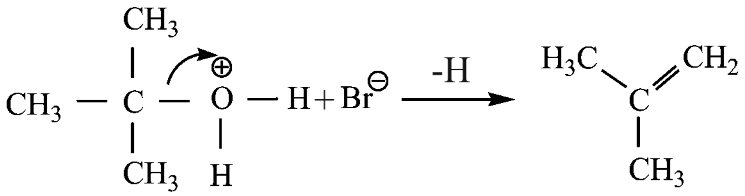
2-methyl-2-propanol

Step-II:



2-bromo-2-methylpropane

Step-III:



2-methylprop-1-ene

(19) Elimination reactions of alcohols

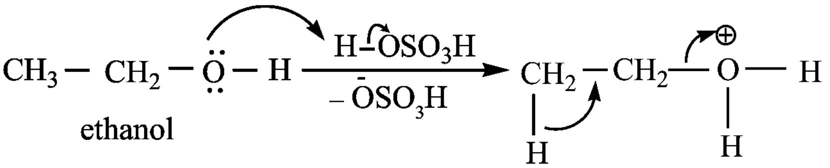
E_2 Mechanism: Example

$$CH_3-CH_2-OH \xrightarrow[443\ K]{H_2SO_4} CH_2=CH_2+H_2O$$

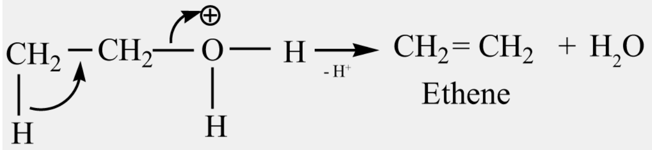
Ethanol Ethene

Mechanism:

Step-I:



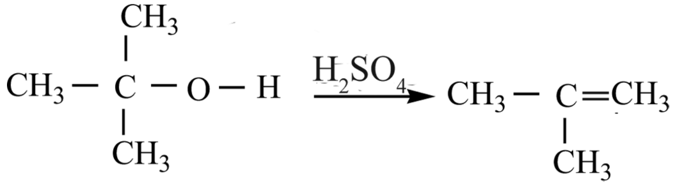
Step-II:



Notes:

(20) E_1 Mechanism: [B° alcohol]

Example

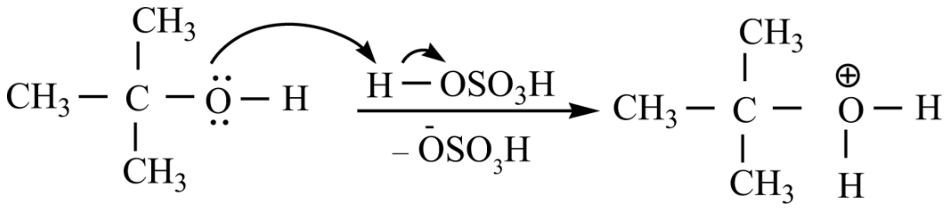


$$CH_3-\overset{CH_3}{\underset{CH_3}{C}}-O-H \xrightarrow{H_2SO_4} CH_3-\underset{CH_3}{C}=CH_3$$

3° butyl alcohol 2-methyl prop-1-one

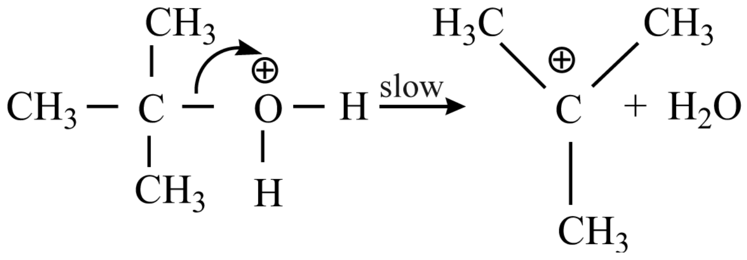
Mechanism:

Step-I:

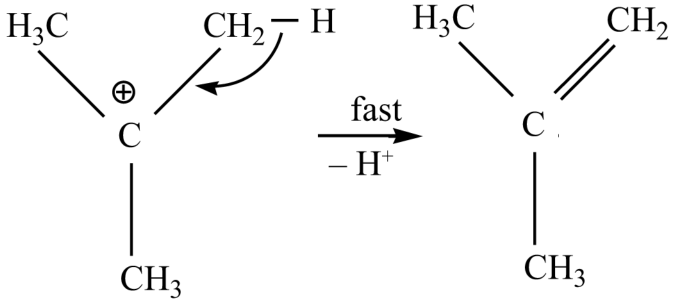


$$CH_3-\overset{CH_3}{\underset{CH_3}{C}}-\ddot{O}-H \xrightarrow[-\bar{O}SO_3H]{H-OSO_3H} CH_3-\overset{CH_3}{\underset{CH_3}{C}}-\overset{\oplus}{\underset{H}{O}}-H$$

Step-II:



Step-III:



2-methylprop-1-ene

Carbonyl compounds and carboxylic acid

Rosenmund Reduction:

$CH_3COCl + H_2 \xrightarrow{Pd/BaSO_4} CH_3CHO + HCl$

Acetyl chloride Acetaldehyde

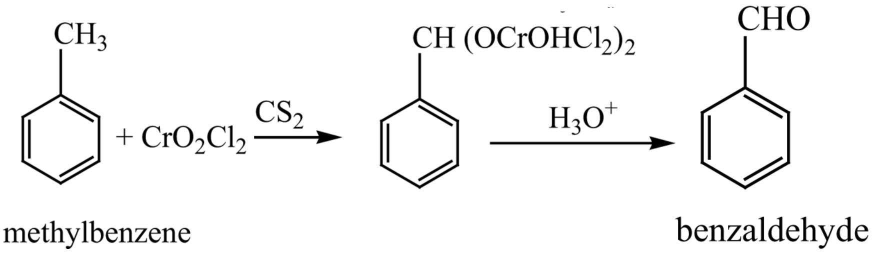
Stephen's reaction:

$CH_3-C\equiv N \xrightarrow[(H)]{SnCl_2/HCl} CH_3-CH=NH$

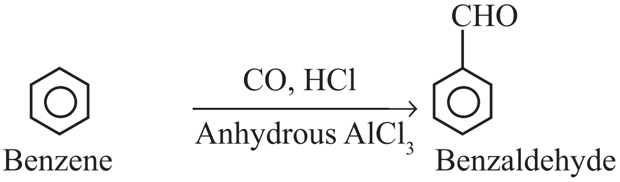
Notes:

$$CH_3-CH{=}NH \xrightarrow{H_3O^+} CH_3-CHO + NH_3$$

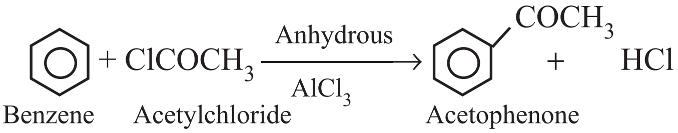
Etard reaction:

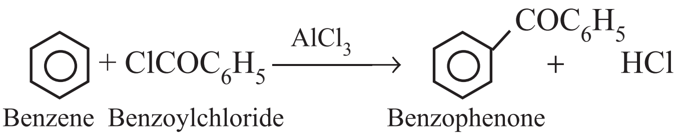


Gattermann-Koch Reaction:



Friedel Crafts acylation:





Reaction with ammonia:

[or preparation of hexamethylene tetramine (or) Urotropine]

$$6HCHO + 4\,NH_3 \longrightarrow (CH_2)_6N_4 + 6\,H_2O$$

Formaldehyde — Hexamethylene tetramine

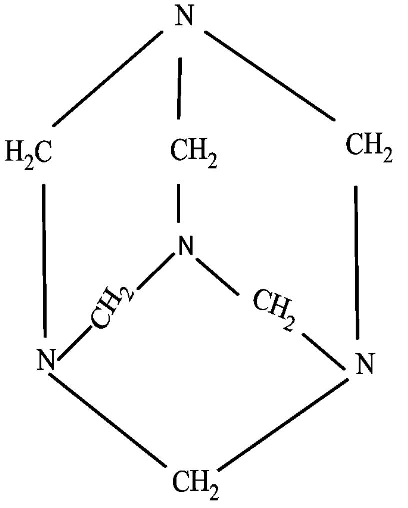
Uses:

(i) Urotropine is used as a medicine to treat urinary infection.

(ii) Nitration of Urotropine under controlled condition gives an explosive RDX (Research and development explosive).

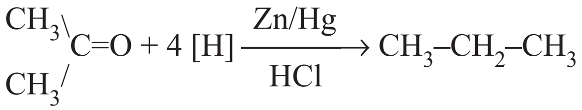
(iii) It is also called cyclonite or cyclotri methylene trinitramine.

Structure:



Hexamethylene tetramine (or) Urotropine

Clemensen reduction:

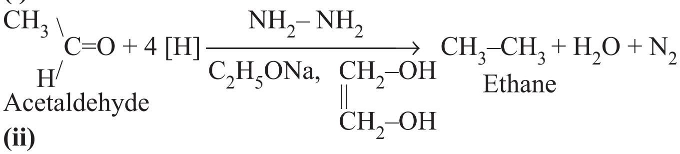


$$\begin{matrix}CH_3\backslash\\ CH_3/\end{matrix}C=O + 4\,[H] \xrightarrow[HCl]{Zn/Hg} CH_3-CH_2-CH_3$$

Acetone Propane

Wolf Kisher reduction: (NH_2 NH_2 -hydrazine)

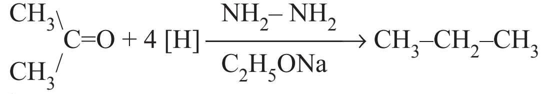
(i)



$$\begin{matrix}CH_3\,\backslash\\ H/\end{matrix}C=O + 4\,[H] \xrightarrow[C_2H_5ONa,\ \begin{matrix}CH_2-OH\\ \|\\ CH_2-OH\end{matrix}]{NH_2-NH_2} CH_3-CH_3 + H_2O + N_2$$

Acetaldehyde Ethane

(ii)

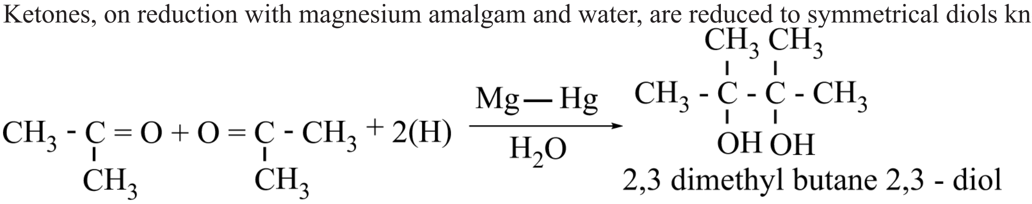


$$\begin{matrix}CH_3\backslash\\ CH_3/\end{matrix}C=O + 4\,[H] \xrightarrow[C_2H_5ONa]{NH_2-NH_2} CH_3-CH_2-CH_3$$

Acetone Propane

Reduction to pinacols:

Ketones, on reduction with magnesium amalgam and water, are reduced to symmetrical diols known as pinacol.



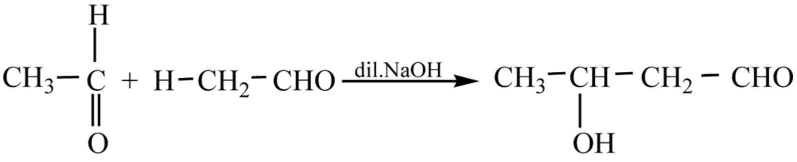
$$CH_3-\underset{CH_3}{\underset{|}{C}}=O+O=\underset{CH_3}{\underset{|}{C}}-CH_3+2(H)\xrightarrow[H_2O]{Mg-Hg}CH_3-\overset{CH_3}{\overset{|}{\underset{OH}{\underset{|}{C}}}}-\overset{CH_3}{\overset{|}{\underset{OH}{\underset{|}{C}}}}-CH_3$$

Acetone Acetone

2,3 dimethyl butane 2,3 - diol
(pinacol)

Aldol condensation:

In presence of dilute base NaOH, or KOH, two molecules of an aldehyde or ketone having **α-hydrogen** add together to give **β-hydroxyl aldehyde** (aldol) or **β-hydroxyl ketone** (ketol). The reaction is called **aldol condensation reaction**.



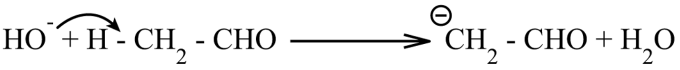
$$CH_3-\overset{H}{\overset{|}{\underset{O}{\underset{\|}{C}}}}+H-CH_2-CHO\xrightarrow{dil.NaOH}CH_3-\underset{OH}{\underset{|}{CH}}-CH_2-CHO$$

Acetaldehyde

Acetaldol
(3 - Hydroxy butanal)

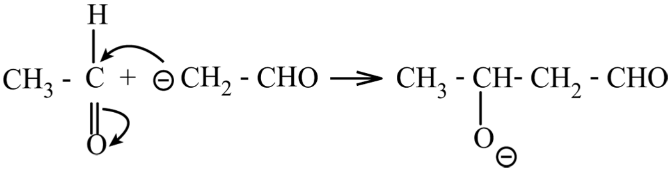
Mechanism:

Step-I:



$$HO^- + H-CH_2-CHO \longrightarrow \overset{\ominus}{C}H_2-CHO+H_2O$$

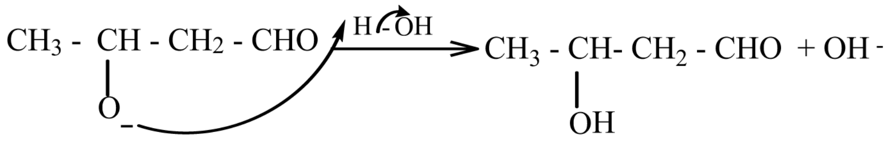
Step-II:



$$CH_3-\overset{H}{\overset{|}{\underset{O}{\underset{\|}{C}}}}+\ominus CH_2-CHO\longrightarrow CH_3-\underset{O^{\ominus}}{\underset{|}{CH}}-CH_2-CHO$$

Notes:

Step-III:



$$CH_3-\underset{\underset{O^-}{|}}{CH}-CH_2-CHO \xrightarrow{H-OH} CH_3-\underset{\underset{OH}{|}}{CH}-CH_2-CHO + OH^-$$

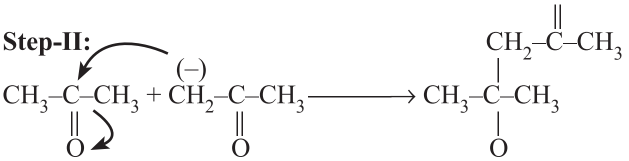
Aldol

Aldol condensation of ketone:

Step-I:

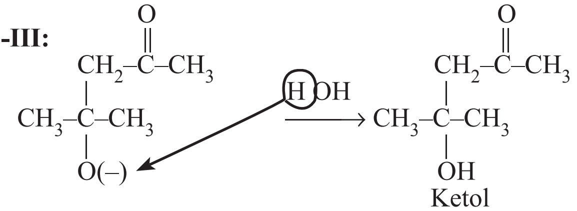
$$\overset{(-)\ H}{CH_3}-CO-CH_3 \xrightarrow{Na\ OH} CH_2-CO-CH_3$$

Step-II:



$$CH_3-\underset{\underset{O}{\|}}{C}-CH_3 + \overset{(-)}{CH_2}-\underset{\underset{O}{\|}}{C}-CH_3 \longrightarrow CH_3-\underset{\underset{O}{|}}{\overset{\overset{CH_2-\overset{\overset{O}{\|}}{C}-CH_3}{|}}{C}}-CH_3$$

Step-III:



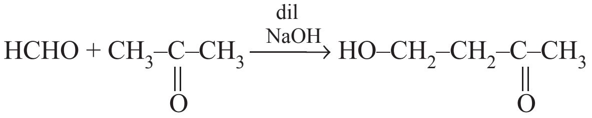
$$CH_3-\underset{\underset{O(-)}{|}}{\overset{\overset{CH_2-\overset{\overset{O}{\|}}{C}-CH_3}{|}}{C}}-CH_3 \xrightarrow{H\ OH} CH_3-\underset{\underset{OH}{|}}{\overset{\overset{CH_2-\overset{\overset{O}{\|}}{C}-CH_3}{|}}{C}}-CH_3$$

Ketol

Crossed aldol condensation:

$HCHO + CH_3CHO \xrightarrow{\text{dil NaOH}} HO-CH_2-CH_2-CHO$

Formaldehyde Acetaldehyde 3-hydroxy propanal



$$HCHO + CH_3-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow[\text{NaOH}]{\text{dil}} HO-CH_2-CH_2-\underset{\underset{O}{\|}}{C}-CH_3$$

Formaldehyde Acetone 4-hydroxy butan 2-one

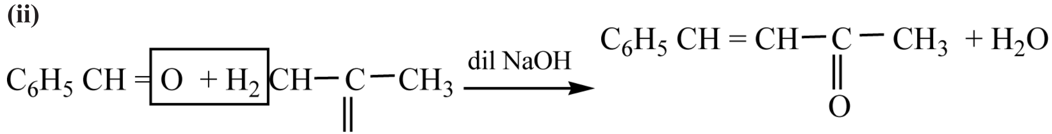
Claisen – Schmidt Condensation:

(i)

$C_6H_5\,CH=\boxed{O + H_2}CH-CHO \xrightarrow{\text{dil NaOH}} C_6H_5\,CH=CH-CHO + H_2O$

Benzaldehyde Acetaldehyde Cinnamaldehyde

(ii)



$$C_6H_5\,CH=\boxed{O + H_2}CH-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{\text{dil NaOH}} C_6H_5\,CH=CH-\underset{\underset{O}{\|}}{C}-CH_3 + H_2O$$

Benzaldehyde Acetone Benzylidene acetone (Benzal acetone)

Cannizaro's reaction:

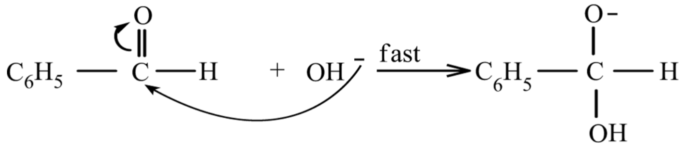
[This reaction is example of disproportionalise reaction]

$C_6H_5CHO + C_6H_5CHO \xrightarrow{50\%\ NaOH} C_6H_5CH_2OH + C_6H_5COONa$

Benzaldehyde Benzyl alcohol Sodium benzoate

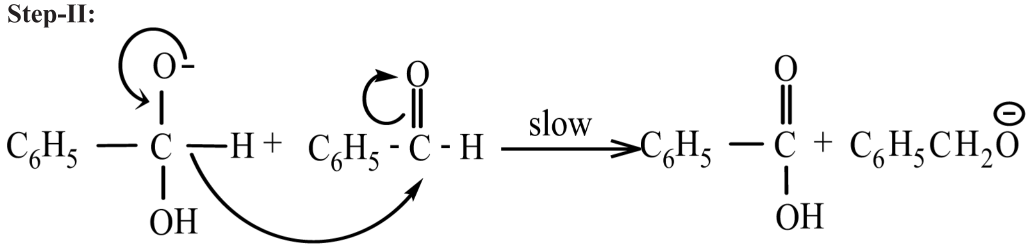
Notes:

Mechanism: Step-I:



$$C_6H_5-\overset{O}{\overset{\|}{C}}-H + OH^- \xrightarrow{fast} C_6H_5-\underset{OH}{\overset{O^-}{C}}-H$$

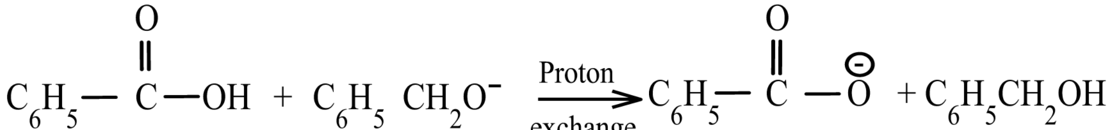
Step-II:



$$C_6H_5-\underset{OH}{\overset{O^-}{C}}-H + C_6H_5-\overset{O}{\overset{\|}{C}}-H \xrightarrow{slow} C_6H_5-\underset{OH}{\overset{O}{\overset{\|}{C}}} + C_6H_5CH_2O^{\ominus}$$

Benzaldehyde

Step-III:



$$C_6H_5-\overset{O}{\overset{\|}{C}}-OH + C_6H_5CH_2O^- \xrightarrow[exchange]{Proton} C_6H_5-\overset{O}{\overset{\|}{C}}-O^{\ominus} + C_6H_5CH_2OH$$

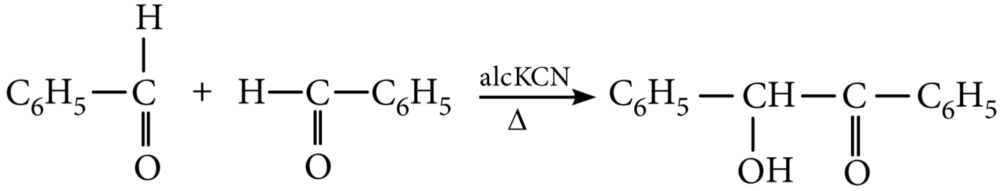
benzoate Benzyl alcohol

Crossed Cannizaro reaction:

$$C_6H_5CHO + HCHO \xrightarrow{NaOH} C_6H_5CH_2OH + HCOONa$$

Benzaldehyde Formaldehyde Benzyl alcohol sodium formate

Benzoin condensation:



$$C_6H_5-\underset{\overset{\|}{O}}{\overset{H}{C}} + H-\underset{\overset{\|}{O}}{C}-C_6H_5 \xrightarrow[\Delta]{alcKCN} C_6H_5-\underset{OH}{CH}-\underset{\overset{\|}{O}}{C}-C_6H_5$$

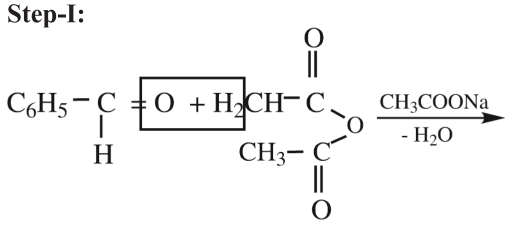
Benzaldehyde

Benzoin

2-hydroxy – 1, 2 – diphenyl ethanone

Perkin's reaction:

Step-I:

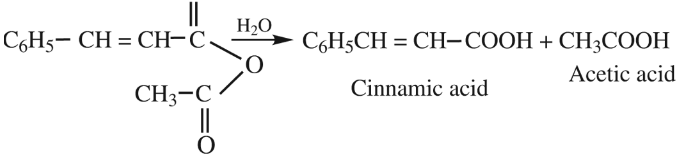


$$C_6H_5-\underset{H}{C}\boxed{=O + H_2}CH-\overset{O}{\overset{\|}{C}}-O-\underset{O}{\underset{\|}{C}}-CH_3 \xrightarrow[- H_2O]{CH_3COONa}$$

Benzaldehyde Acetic anhydride

Notes:

Step-II:

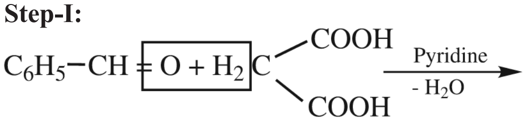


Cinnamic acid

Acetic acid

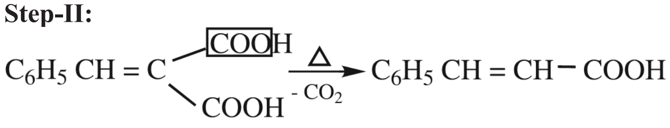
Knoevenagal reaction:

Step-I:



Benzaldehyde Malonic acid

Step-II:



Cinnamic acid

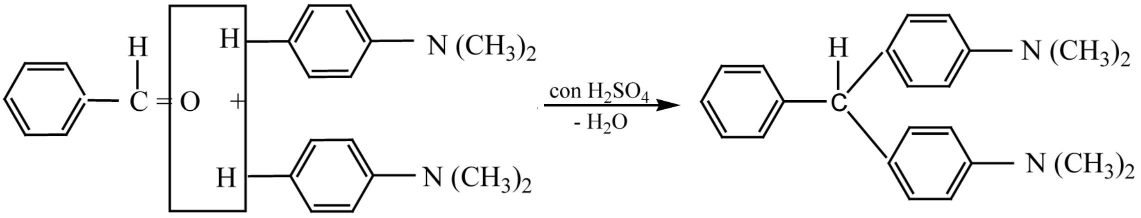
Formation of schiff's base: [Reaction with ammonia]

$$C_6H_5-CH\boxed{=O+H_2}N-C_6H_5 \xrightarrow[\Delta]{H^+} C_6H_5-CH=N-C_6H_5 + H_2O$$

Benzaldehyde Aniline

Benzal aniline
(Schiff's base)

Malachite green dye preparation:

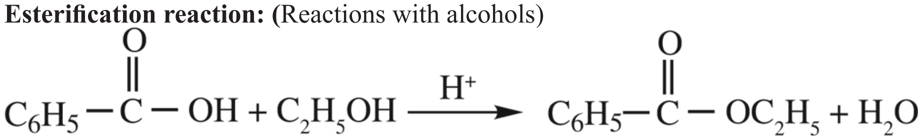


Benzaldehyde

N, N - Dimethyl aniline

Malachite green dye

Esterification reaction: (Reactions with alcohols)

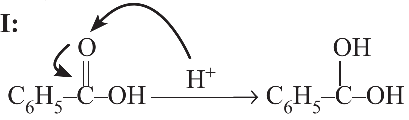


$$C_6H_5-\overset{\overset{\displaystyle O}{\|}}{C}-OH + C_2H_5OH \xrightarrow{H^+} C_6H_5-\overset{\overset{\displaystyle O}{\|}}{C}-OC_2H_5 + H_2O$$

Benzoic acid ethyl benzoate

Mechanism:

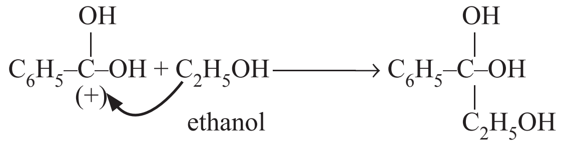
Step-I:



Benzoic acid

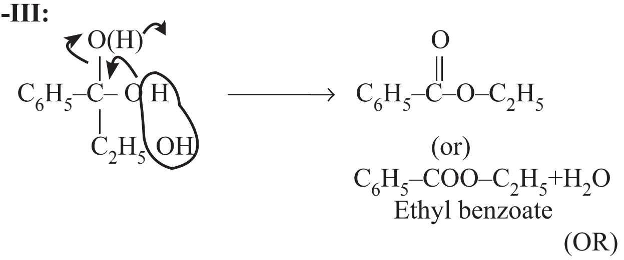
Notes:

Step-II:



$$C_6H_5-\overset{\overset{OH}{|}}{\underset{(+)}{C}}-OH + \underset{\text{ethanol}}{C_2H_5OH} \longrightarrow C_6H_5-\overset{\overset{OH}{|}}{\underset{\underset{C_2H_5OH}{|}}{C}}-OH$$

Step-III:



$$C_6H_5-\overset{\overset{O(H)}{|}}{\underset{\underset{C_2H_5OH}{|}}{C}}-OH \longrightarrow C_6H_5-\overset{\overset{O}{\|}}{C}-O-C_2H_5$$

(or)

$C_6H_5-COO-C_2H_5+H_2O$

Ethyl benzoate

(OR)

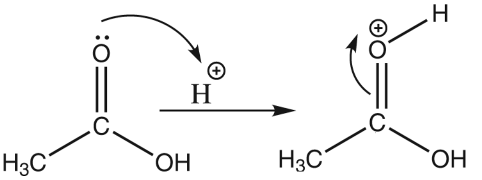
Esterification reaction: (Mechanism)

E.g.

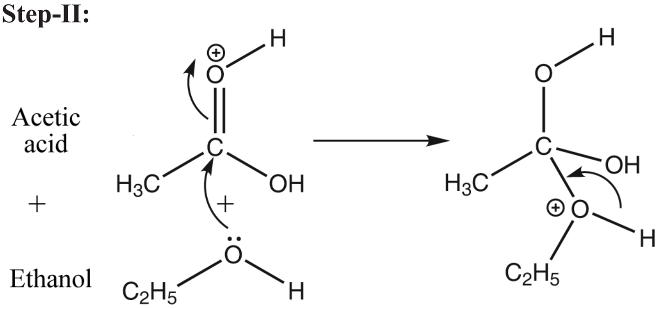
$$\underset{\text{Acetic acid}}{CH_3COO}\boxed{H} + C_2H_5\boxed{OH} \longrightarrow \underset{\text{ethyl ethanoate}}{CH_3COOC_2H_5}$$

Mechanism:

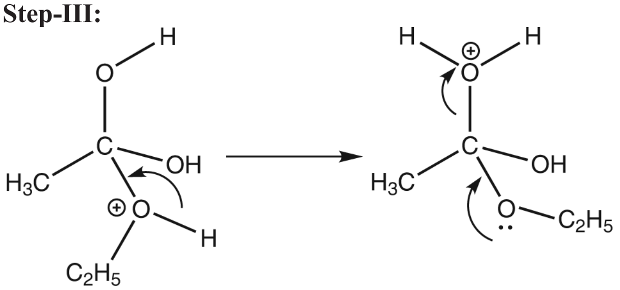
Step-I:



Step-II:

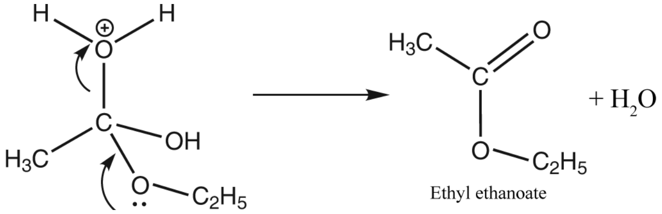


Step-III:



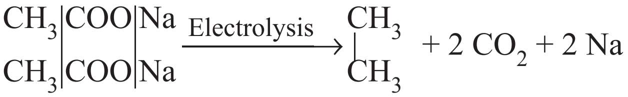
Notes:

Step-IV:



Kolbe's electrolytic decarboxylation: (Kolbe's electrolysis)

Electrolysis of concentrated aqueous solution of sodium salt of acids gives hydrocarbon.

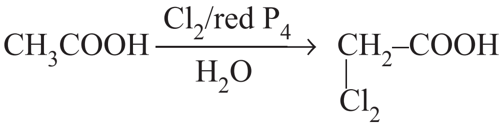


$$\begin{array}{l|l|l} CH_3 & COO & Na \\ CH_3 & COO & Na \end{array} \xrightarrow{\text{Electrolysis}} \begin{array}{l} CH_3 \\ | \\ CH_3 \end{array} + 2\,CO_2 + 2\,Na$$

Sodium acetate Ethane

HVZ - Hell-Volhard – Zelinsky (HVZ) reaction:

The halogenation in α carbon atom is carried out with halogen and phosphorus trihalide, the reaction is HVZ reaction.



$$CH_3COOH \xrightarrow[H_2O]{Cl_2/\text{red } P_4} \begin{array}{l} CH_2–COOH \\ | \\ Cl_2 \end{array}$$

Acetic acid Mono chloro acetic acid

Silver Mirror test:

$$HCOO^- + 2Ag^+ + 3OH^- \longrightarrow 2\,Ag + CO_3^{2-} + 2H_2O$$

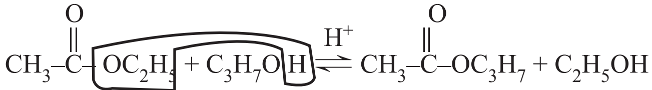
(Tollens reagent) Silver mirror

Reducing action of formic acid:

$$HCOO^- + 2Cu^{2+} + 5OH^- \longrightarrow Cu_2O + CO_3^{2-} + 2H_2O$$

Red ppt

Transesterification:

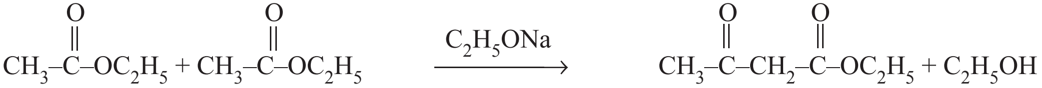


$$CH_3–\overset{\overset{O}{\|}}{C}–\boxed{OC_2H_5} + C_3H_7O\boxed{H} \overset{H^+}{\rightleftharpoons} CH_3–\overset{\overset{O}{\|}}{C}–OC_3H_7 + C_2H_5OH$$

Ethyl acetate Propyl alcohol Propyl acetate Ethyl alcohol

Claisen condensation:

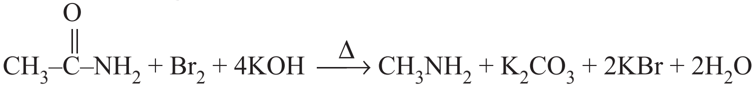
Esters containing atleast one α-hydrogen atom undergo self condensation in the presence of a strong base such as sodium ethoxide to form β-**keto ester**.



$$CH_3–\overset{\overset{O}{\|}}{C}–OC_2H_5 + CH_3–\overset{\overset{O}{\|}}{C}–OC_2H_5 \xrightarrow{C_2H_5ONa} CH_3–\overset{\overset{O}{\|}}{C}–CH_2–\overset{\overset{O}{\|}}{C}–OC_2H_5 + C_2H_5OH$$

Ethyl acetate Ethyl acetate Ethyl aceto acetate Ethyl alcohol

Hoffmann's degradation:



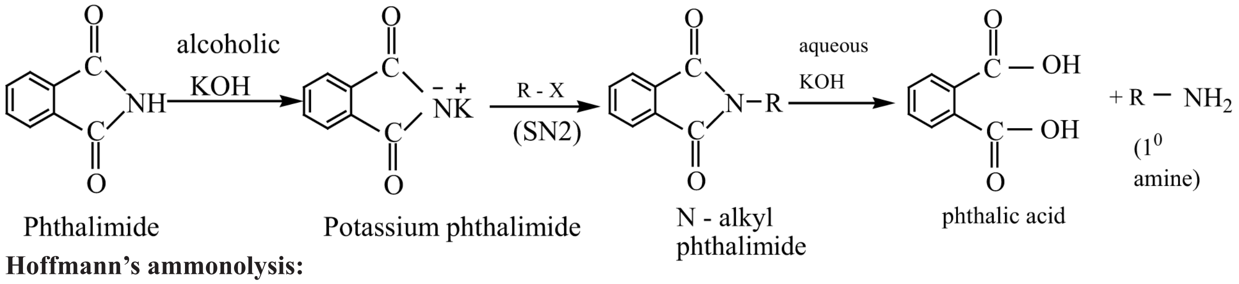
$$CH_3–\overset{\overset{O}{\|}}{C}–NH_2 + Br_2 + 4KOH \xrightarrow{\Delta} CH_3NH_2 + K_2CO_3 + 2KBr + 2H_2O$$

Acetamide Ethyl amine

Notes:

ORGANIC NITROGEN COMPOUNDS

(i) Gabriel phthalimide synthesis:



Hoffmann's ammonolysis:

Step-I:

$$CH_3-Br \xrightarrow[\Delta\ -HBr]{\ddot{N}H_3} CH_3-\ddot{N}H_2 \xrightarrow[-HBr]{CH_3-Br} (CH_3)_2\ddot{N}H$$

Methyl Bromide

Step-II:

$$(CH_3)_2\ddot{N}H \xrightarrow[-HBr]{CH_3Br} (CH_3)_3\ddot{N}$$

2^0 - amine　　3^0- amine

Step-III:

$$(CH_3)_3\ddot{N} \xrightarrow{CH_3Br} (CH_3)_4\overset{+}{N}Br^-$$

3^0- amine　　Quarternary ammonium bromide

Sabatier – Mailhe method:

(Ammonolysis of hydroxyl compounds)

When vapour of an alcohol and ammonia are passed over alumina, W_2O_5 (or) silica at 400°C, all types of amines are formed. This method is called **Sabatier – Mailhe** method.

Step-I: $$C_2H_5OH \xrightarrow[Al_2O_3,\ -H_2O]{\ddot{N}H_3} C_2H_5-\ddot{N}H_2$$

Ethyl alcohol　　Ethyl amine (1° amine)

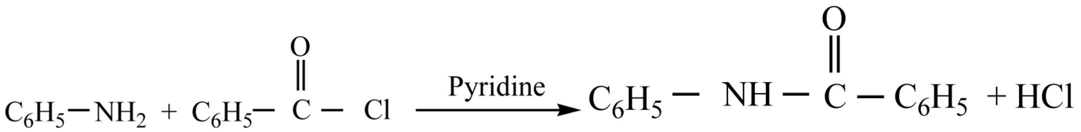
Step-II: $$C_2H_5-\ddot{N}H_2 + C_2H_5OH \longrightarrow (C_2H_5)_2\ddot{N}H + H_2O$$

Diethyl amine (2°amine)

Step-III: $$(C_2H_5)_2\ddot{N}H + C_2H_5OH \longrightarrow (C_2H_5)_3\ddot{N} + H_2O$$

Triethyl amine (3°amine)

Schotten Baumaan reaction:

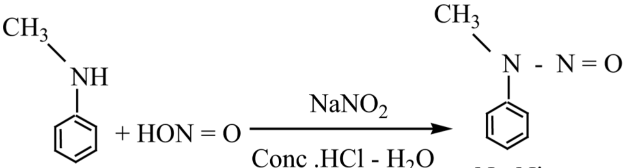


$$C_6H_5-NH_2 + C_6H_5-\overset{\overset{O}{\|}}{C}-Cl \xrightarrow{Pyridine} C_6H_5-NH-\overset{\overset{O}{\|}}{C}-C_6H_5 + HCl$$

Aniline　　Benzoylchloride　　N - phenyl benzamide

Notes:

Libermann's nitroso test: (Yellow oil - preparation)



N - methylaniline

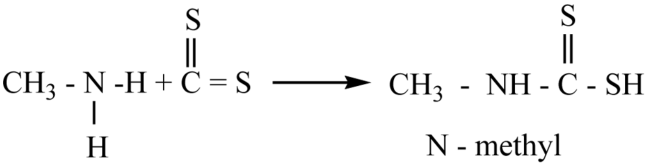
N - Nitroso methyl
phenyl amine
(yellow oil)

Carbylamine reaction: [Identify - 1° amino test]

$$\underset{\text{Ethylamine}}{C_2H_5NH_2} + \underset{\text{Chloroform}}{CHCl_3} + 3KOH \longrightarrow \underset{\text{Ethylisocyanide}}{C_2H_5\text{-}NC} + 3KCl + 3H_2O$$

Mustard oil reaction

(i) Step-I:

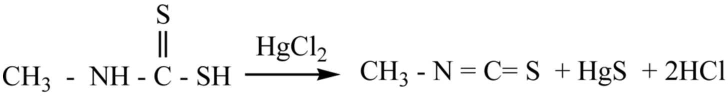


$$CH_3 - \underset{\underset{H}{|}}{N} - H + \overset{\overset{S}{\|}}{C} = S \longrightarrow CH_3 - NH - \overset{\overset{S}{\|}}{C} - SH$$

Methylamine

N - methyl
dithiocarbamic acid

Step-II:



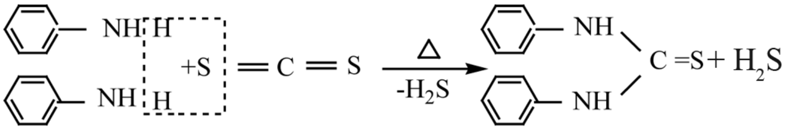
$$CH_3 - NH - \overset{\overset{S}{\|}}{C} - SH \xrightarrow{HgCl_2} CH_3 - N = C = S + HgS + 2HCl$$

N - methyl
dithiocarbamic acid

Methyl
isothiocyanate
(Mustard oil smell)

(OR)

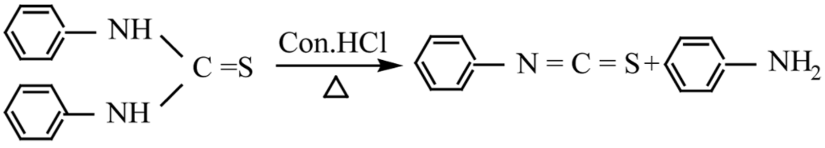
Step-I:



Aniline

S - diphenyl
thiourea

Step-II:

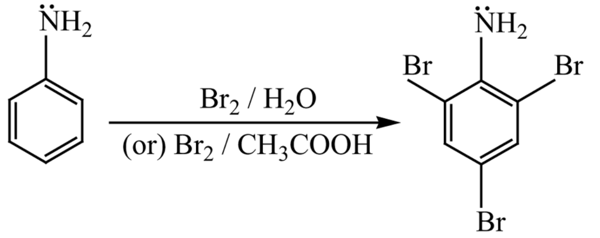


S - diphenyl
thiourea

Phenyl isothiocyanate

Notes:

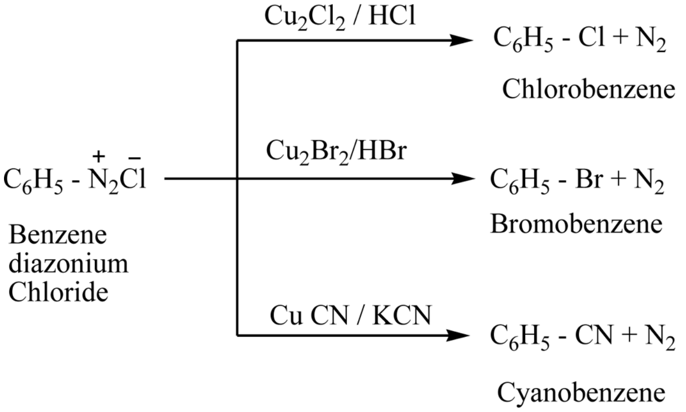
Bromination:



Aniline

2,4,6, tribromo aniline
(white precipitate)

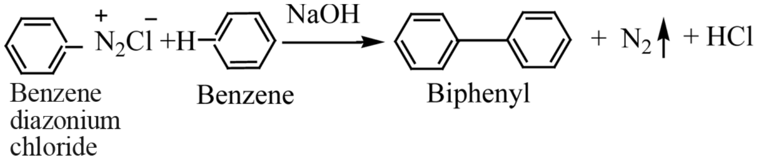
Sandmeyer reaction:



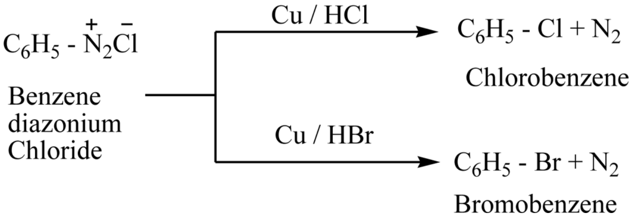
Why aniline does not undergo Friedel-Crafts reaction:

It does not under go Friedel-Crafts reaction (alkylation and acetylation) we know aniline is basic in nature and it donates its lone pair to the lewis acid $AlCl_3$ to form an adduct which inhibits further the electrophilic substitution reaction.

Gomberg reaction:



Gattermann reaction:



Notes:

Baltz – schiemann reaction:

Step-I:

$$C_6H_5 - \overset{+}{N_2}\overset{-}{Cl} + HBF_4 \longrightarrow C_6H_5 - \overset{+}{N_2}\overset{-}{BF_4} + HCl$$

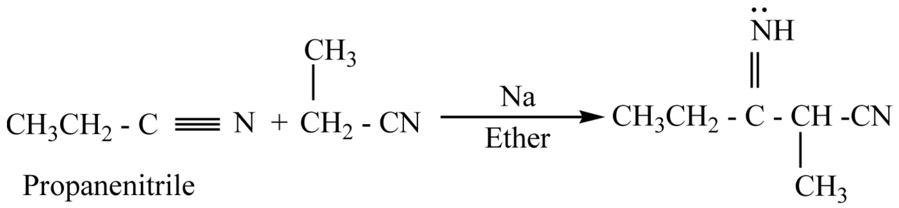
Fluoroboric acid Benzenediazonium fluoroborate

Step-II:

$$C_6H_5 - \overset{+}{N_2}\overset{-}{BF_4} + NaNO_2 \xrightarrow{\triangle} C_6H_5 - F + BF_3 + N_2$$

Benzenediazonium fluoroborate Fluorobenzene

Thorpe nitrile condensation:

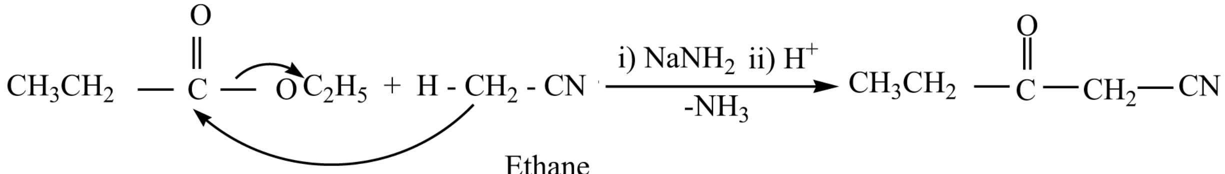


$$CH_3CH_2 - C \equiv N + \underset{}{\overset{CH_3}{CH_2}} - CN \xrightarrow[\text{Ether}]{\text{Na}} CH_3CH_2 - \overset{\ddot{N}H}{\overset{\|}{C}} - \underset{CH_3}{CH} - CN$$

Propanenitrile

3 - imino - 2- methyl pentanenitrile

Levine and Hauser acetylation:



$$CH_3CH_2 - \overset{O}{\overset{\|}{C}} - OC_2H_5 + H - CH_2 - CN \xrightarrow[-NH_3]{\text{i) }NaNH_2\text{ ii) }H^+} CH_3CH_2 - \overset{O}{\overset{\|}{C}} - CH_2 - CN$$

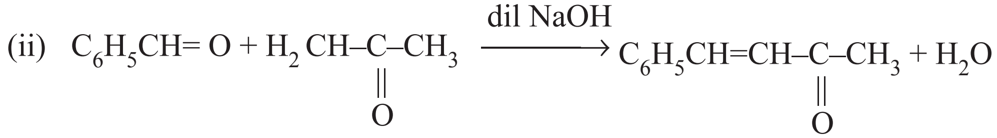
Ethyl Propionate Ethane nitrile 3 - Ketopentanenitrile

$+ C_2H_5OH$

Claisen-Schmidt reaction:

(i) $C_6H_5CH{=}O + H_2CH–CHO \xrightarrow{\text{dil NaOH}} C_6H_5CH{=}CH–CHO + H_2O$

Benzaldehyde Acetaldehyde Cinnamaldehyde



(ii) $C_6H_5CH{=}O + H_2CH–\overset{}{\underset{O}{\underset{\|}{C}}}–CH_3 \xrightarrow{\text{dil NaOH}} C_6H_5CH{=}CH–\underset{O}{\underset{\|}{C}}–CH_3 + H_2O$

Benzaldehyde Acetone Benzylidene acetone

(Benzal acetone)

Notes:

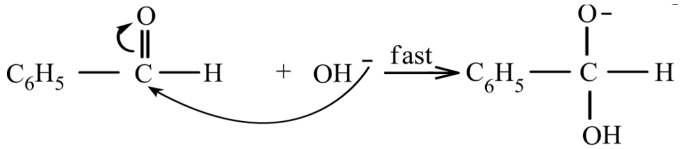
Cannizaro's reaction:

$$C_6H_5CHO + C_6H_5CHO \xrightarrow{50\%\ NaOH} C_6H_5CH_2OH + C_6H_5COONa$$

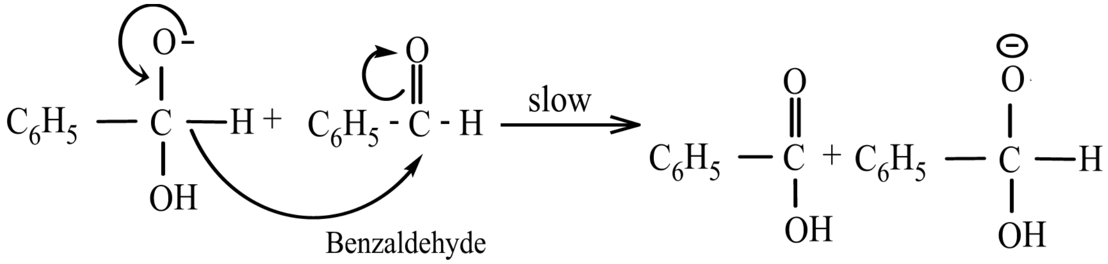
Benzyl alcohol Sodium benzoate

Mechanism:

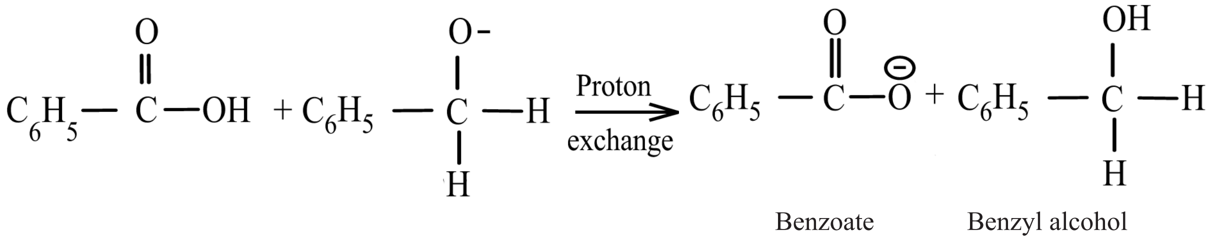
Step-I:



Step-II:



Step-III:



Williamsons synthesis:

Overall reactions:

$$CH_3\text{-}ONa + Br\text{-}C_2H_5 \xrightarrow{\Delta} CH_3\text{-}O\text{-}C_2H_5 + NaBr$$

Mechanism:

$$CH_3-O^-Na^+ + CH_3-CH_2-Br \xrightarrow[\Delta]{-NaBr} CH_3-CH_2-\ddot{\underset{\cdot\cdot}{O}}-CH_3$$

Sodium methoxide Ethyl bromide methoxyethane

****

Notes: